Neoplatonic Logic of Relations

How Logic Can Abet Mysticism

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Neoplatonism (Plotinus, 3rd Century)

Mystical Philosophy:

- All reality emanates from the One in "hypotheses" or levels of Being
- The Principle Hypotheses: One, Being (Ideas), Spirit, Matter
- Knowledge is remotion to the One, resulting in mystical ineffable enlightenment.

Neoplatonists who wrote and lectured on Aristotelian logic:

Porphyry (*Isagoge*, 3rd century)
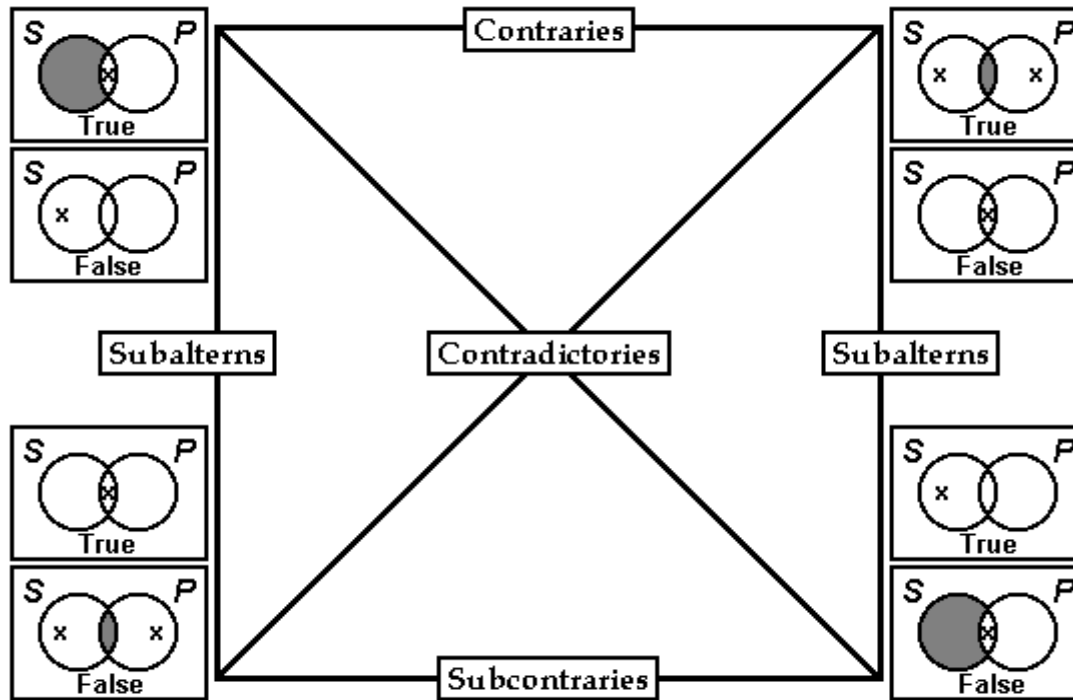
Proclus, Ammonius, Philoponus (5th century)

Aristotlian Logic has Non-Relational, Subject-Predicate Propositions

A: All S are P

$\forall x(Sx \rightarrow Px)$ or $\sim\exists x(Sx \wedge \sim Px)$

E: No S is P

$\forall x(Sx \rightarrow \sim Px)$ or $\sim\exists x(Sx \wedge Px)$

I: Some S are P

$\exists x(Sx \wedge Px)$ or $\sim\forall x(Sx \rightarrow \sim Px)$

O: Some S are not P

$(\exists x(Sx \wedge \sim Px)$ or $\sim\forall x(Sx \rightarrow Px)$

The Syllogistic's Subject-Predicate Propositions cannot express relations

- **Proposal 1.**   aRb   **resolves to**   Fa $\wedge$ Fb (for some F)
- **Theorem: All relations are symmetric** . aRb $\rightarrow$ bRa
- **Proof**: Suppose aRb. Then by def Fa $\wedge$ Fb. Then Fb $\wedge$ Fa. Then by def bRa. QED.

- **Proposal 2.**   aRb   **resolves to**   Fa $\wedge$ Gb (for some F and G)
- **Theorem: All relations are transitive**   (aRb $\wedge$ bRc) $\rightarrow$ aRc
- **Proof.** Assume aRb and bRc. The by def Fa $\wedge$ Gb, and Fb $\wedge$ Gc. Then Fa $\wedge$ Gb. Then by def aRb. QED.

Relational Propositions in Geometry

Euclid, *Elements*, Proposition 19, Book I (Heath translation)

In any triangle the side opposite the greater angle is greater.

- Let *ABC* be a triangle having the angle *ABC* greater than the angle *BCA.* I say that the side *AC* is greater than the side *AB.* If not, either *AC* equals *AB* or it is less than it. Now *AC* does not equal *AB,* for then the angle *ABC* would equal the angle *ACB,* but it does not. Therefore *AC* does not equal *AB.* Neither is *AC* less than *AB,* for then the angle *ABC* would be less than the angle *ACB,* but it is not. Therefore *AC* is not less than *AB.* And it was proved that it is not equal either. Therefore *AC* is greater than *AB.* Therefore *in any triangle the side opposite the greater angle is greater.* QED.

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- Herlinus and Dasypodious (1566), Reformulation of Euclid's proof into Syllogisms

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- **First Exposition**

- Major: Every triangle with equal sides has the angles at the base that are equal (Prop 5)

- Minor: Triangle αβγ has a side αγ equal to αβ. (hypothesis)

- Conclusion   In triangle αβγ, angle αβγ is equal to αγβ.

- **Second Exposition**

- Major: Every triangle with equal sides has the angles at the base that are equal. (Prop 5)

- Minor: the angle αβγ is not equal to angle αβγ.   (hypothesis)

- Conclusion   αγ is not equal to side αβ.

- In the triangle αβγ, let the side latus αγ be less than the side αβ.

- **Third Exposotion.**

- Major: In any triangle the angle opposite the greater side is greater. (Prop 18)

- Minor: In triangle αβγ, side αβ is greater than side αγ. (hypothesis)

- Conclusion: in triangle αβγ, angle βγα, is greater than angle αβγ.

- **Fourth Exposition**

- Major: If in triangle αβγ, side αγ is less than side αβ, the angle αβγ is less than angle αγβ. (third syllogism above)

- Minor: In triangle αβγ, angle αβγ is not less than angle αβγ. (hypothesis)

- Conclusion: In triangle αβγ, side αγ is not less than side αβ.

- **Fifth Exposition**

- Major: In the triangle αβγ, αγ is equal to, greater than or less than αβ. (*per se* true)

- Minor: αγ is not equal to, greater than or less than αβ. (second syllogism above)

-                 αγ is not less than αβ. (fourth syllogism above)

- Conclusion: triangle αβγ, side αγ is greater than side αβ.

Details of First Step. Whatever triangle has two equal sides also has two angles, which adjoin two equal lines. Triangle αβγ has a side αγ equal to αβ. *Ergo*, the angle αβγ is equal to the angle αγβ.

Exposition as a Syllogism

- Major: Every triangle with equal sides has the angles at the

    base that are equal (Prop 5)

- Minor: Triangle αβγ has a side αγ equal to αβ. (hypothesis)

- Conclusion In triangle αβγ, angle αβγ is equal to αγβ.

Rendition into First-Order Logic:

$\forall xyz(T(xyz) \wedge E(xzxy). \rightarrow E(xyzxzy))$

$\underline{T(\alpha\beta\gamma) \wedge E(\alpha\gamma\alpha\beta)}$

$\therefore T(\alpha\beta\gamma) \wedge E(\alpha\beta\gamma\alpha\alpha\beta)$

Proclus, *Elements of Theology*, 5th Century

Ontic Order described by Comparative Adjectives:

…the **higher** cause (*aitiotern*), being the **more efficacious** (*drastikteron*), operates **sooner** upon the participant (for where the same thing is affected by two causes it is affected first by the **more powerful** (*dunatteron*); and in the activity of the secondary the **higher** is co-operative, because all the effects of the secondary are concomitantly generated by the **more determinative** cause (*aititeron*). … .All those characters which in the originative causes have **higher** (*huperteran*) and **more universal** (*holikteron*) rank become in the resultant beings, through the irradiations which proceed from them, a kind of substratum for the gifts of the **more specific** principles (*merikteron*).

Whatever principle is the cause of the **greater** (*pleionen*) number of effects is **superior** (*kreitton*) to that which has a power limited to **fewer** (*elattona*) objects and which gives rise to parts of those existents constituted by others as wholes.

For if the one is cause of **fewer** (*elattonn*) effects, the other of **more** (*pleionn*), and the fewer form a part of the more numerous, then whatever is produced by the former cause will be produced also by the latter,...The latter is therefore **more powerful** (*dunatteron*) and **comprehensive** (*perilptikteron*) ..., and that which can give rise to **more** (*plei*) effects has **greater** (meizona) and **more universal** (*olikteran*) power. But this means that it is **nearer** (*egguter*) to the cause of all things; and what is **nearer** (*egguter*) to the cause is in the **greater** (*meizons*) measure of good, the Good being that cause. The cause of the **more numerous** (*pleionn*) effects is therefore **superior** (*kreitton*) in its being to that which produces **fewer** (*elattona*).

Mass Nouns, with their families of Comparative and Scalar Adjectives

Ordinary Language Examples:

- *Happiness: happier than; ecstatic, happy, content, so-so, down, sad, miserable*
- *Heat: hotter than; boiling, hot, warm, tepid, cool, cold, freezing*
- *Hardness: harder than; adamantine, hard, solid, firm, tangible, weak, wispy*
- *Solidity: stronger than; strong, solid, self-supporting, rickety, dangerous*
- *Attention: more attentive than; riveted, attentive, awake, wandering, dreamy, asleep*
- *Lucidity: more incisive than; incisive, lucid, cognizant, scatter-brained, demented*
- *Intelligence: smarter than; brilliant, smart, pedestrian, dull, stupid*
- *Bravery: braver than; heroic, brave, dutiful, cowardly, craven*

Neoplatonic Examples:

- *Being: more real than absolute, substantial, subsistent, insubstantial, unreal*
- *Universality: more common than; all, most, some, rare, unheard of*
- *Temporality: more lasting than; eternal, occasional, never*
- *Necessity: truer than; necessary, probable, possible, improbable, impossible*
- *Virtue: better than; supererogatory, good, neutral, bad, evil*

Laurence Horn, *Natural History of Negation,* Scalar "test frames",

- X is not only P_2, but P_1.
- X is at least P_2, if not (downright) P_1.
- X is P_2, {or/possibly} even P_1.
- X is not even P_2, {let alone/much less} P_1.
- X is P_2, and is {in fact/indeed} P_1.
- X is P_1, or at least P_2.

Comparative Adjective	Scalar Series
• *happier than:*	*ecstatic, happy, content, so-so*
• *sadder than:*	*miserable, sad, down, so-so*
• *hotter than:*	*boiling, hot, warm, tepid*
• *colder than:*	*freezing, cold, cool, tepid*

The Happiness Scale

hyper-happy happy   not-happy   un-happy

1 …. *ecstatic   happy   content so-so   discontent   sad   miserable* …. 0

m   **n**   *e*   **-n**   **-m**

- Positive and negative **intensifiers** on scalar adjectives (**hyper**, **super, really; huper, sub**, *less*)
- Relations expressed via intensifiers (*John is smart, but Mary is* **really/super** *smart*)

Proclus, *Elements of Theology*, Hyper and Privative Negation

- ...not-Being has a **number of senses**, **one superior to Being**, **another which is of the same rank as Being**, **and yet another which is the privation of Being**, it is clear, surely that we can postulate also **three types of negation, one superior to assertion, another inferior to assertion, and another in some way equally balanced by assertion.**

- In truth my view is that **negations come in three sorts,** one sort is for beings of a form **more fundamental than affirmations**. These are generative and perfective of those things generated in affirmation. Another type is placed at the **same level as affirmations**, and here affirmation is not in any way more worthy than negation. Finally, there are those with a nature **inferior to affirmations**, namely **privations** of affirmations
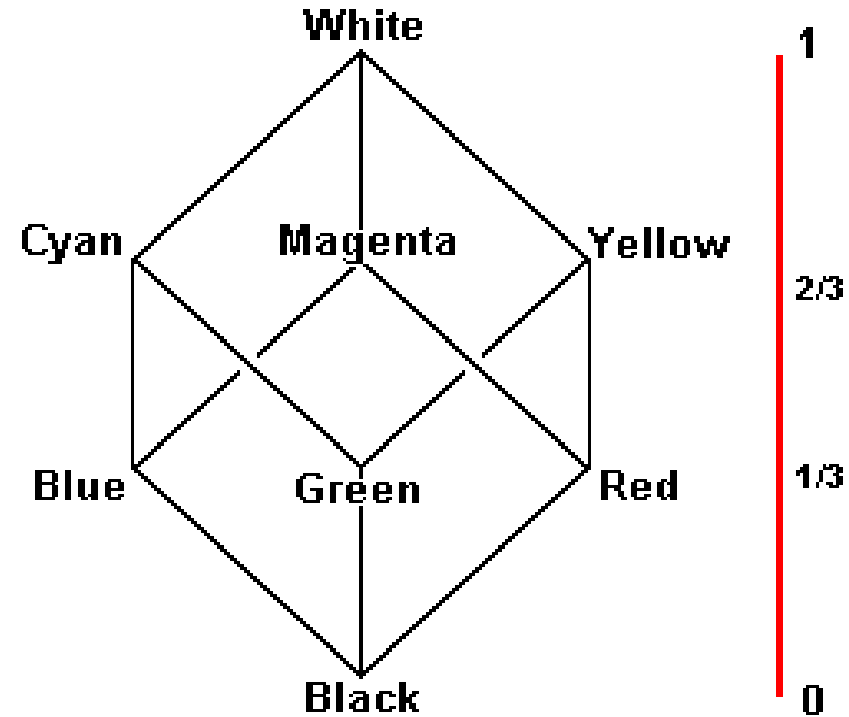
Causal Extension of nodes in the Ontic Hierarchy.
Degrees of Being fall in the inverse order.
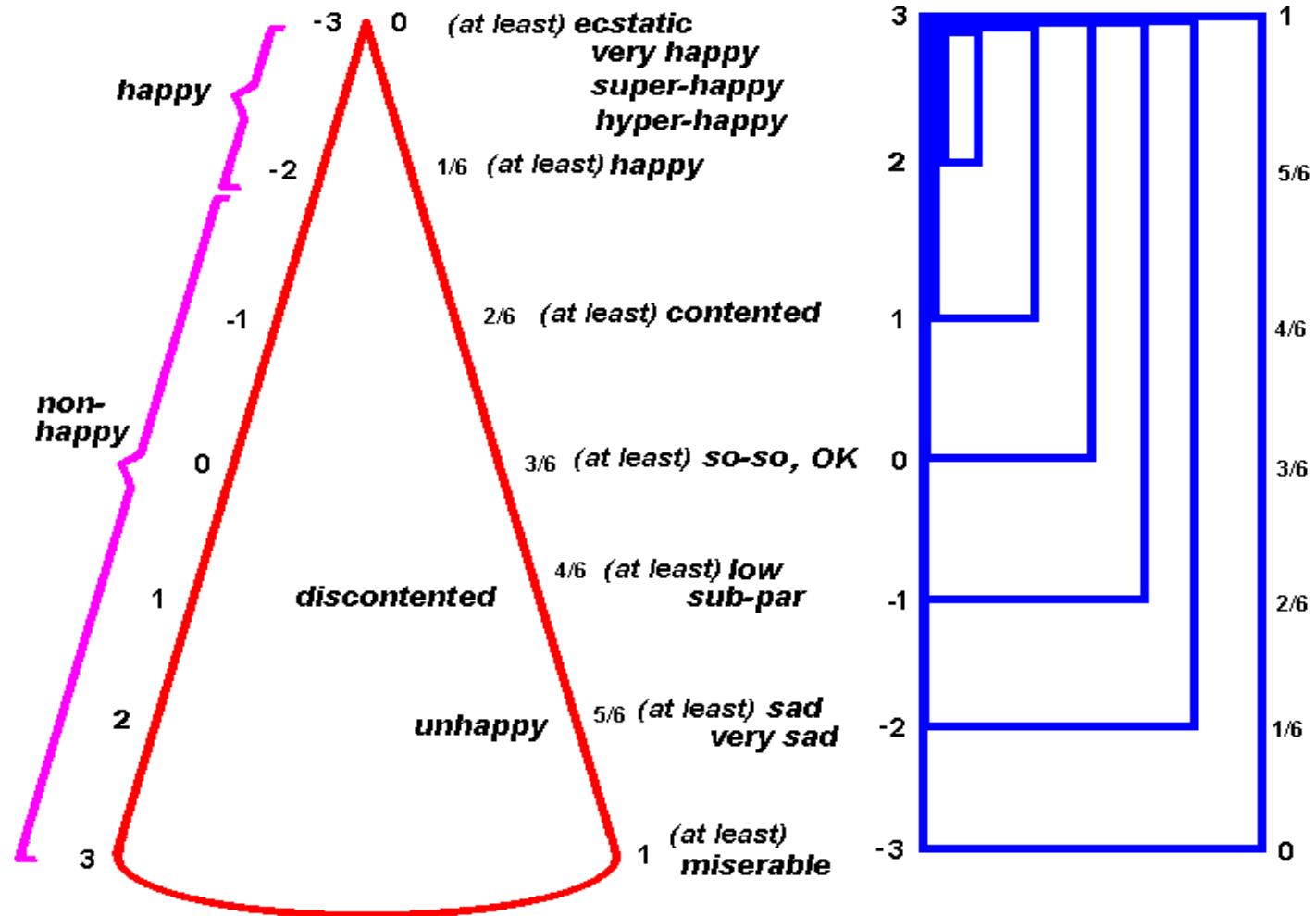
Privative Degrees of White Light Generate of Distinct Properties
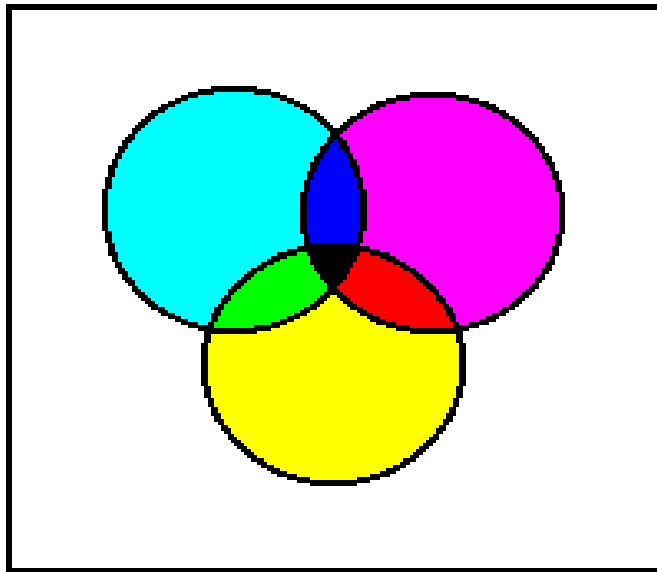
Filters Over White Light

Boolean Algebra of Light Privation

The Algebra of Light

Proclus, Remotion to the One via Hypernegation

- The One = ~ (The One in Being)
- The One in Being = ~ (The Intelligible Whole)
- The Intelligible Whole = ~ (The Intelligible Many)
- The Intelligible Many = ~ (The Intelligible Number)
- The Intelligible Number = ~ (The Composite)
- The Composite = ~ Shape
- Shape = ~ (The In-Itself and In-Others)
- The In-Itself and In-Others = ~ (The In-Motion and At-Rest)
- The In-Motion and At-Rest = ~ (The Same and Different)
- The Same and Different = ~ (The Like and Unlike)
- The Like and Unlike = ~ (The Touching and Not Touching)
- The Touching and Not Touching

Proclus, Iterative Application of Hypernegation

- The One   =   ~ (The One in Being)
-             =   ~~ (The Intelligible Whole)
-             =   ~~~ (The Intelligible Many)
-             =   ~~~~ (The Intelligible Number)
-             =   ~~~~~ (The Composite)
-             =   ~~~~~~ Shape
-             =   ~~~~~~~ (The in Itself and Others
-             =   ~~~~~~~~ (The In Motions and Rest)
-             =   ~~~~~~~~~ (The Same and Different)
-             =   ~~~~~~~~~~ (The Like and Unlike)
-             =   ~~~~~~~~~~~ (The Touching and Not   Touching)
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Proclus' Neoplatonic Contrariety: Can Be Jointly True

- But **contraries** in the Heavens naturally coexist. The motion of the Same is contrary to the motion of the Other, but the same thing (the heavens) is moved in both ways, and when it is moving in one way, it does not abandon the other motion.

- But the **contraries** in Intellect, being unified to the highest degree, partless and immaterial, and constituted as a single form, are creative in company with one another….In sum the contraries in Matter flee one another, those in the heavens co-exist.
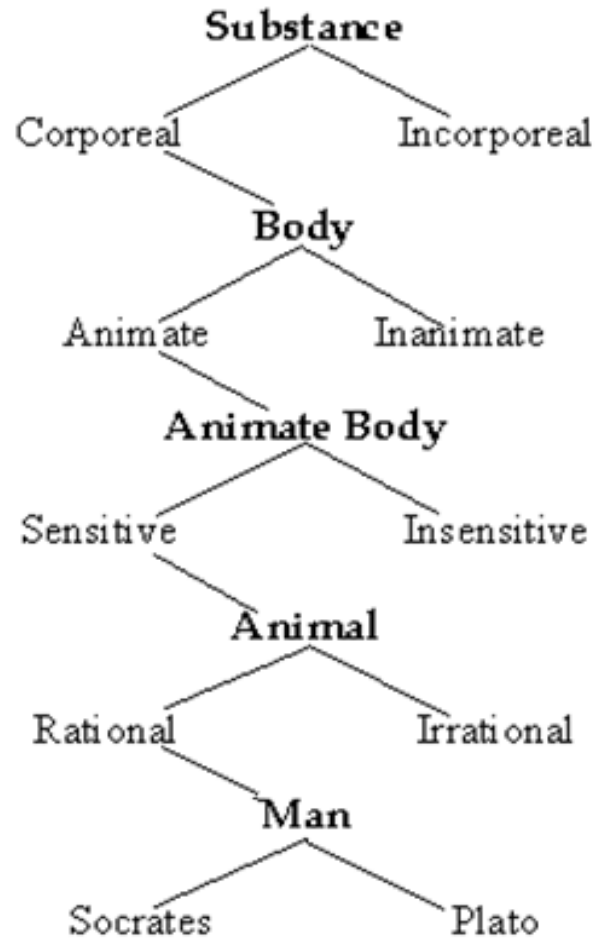
Non-Classical Logical Properties of Ontic Inclusion, Hyper and Privative Negation

Let $\leq$ be ontic causation, the converse of causal extension inclusion

- $\neg\neg x \leq \neg x \leq x \leq \sim x \leq \sim\sim x$
- $x \leq y \rightarrow \sim x \leq \sim y$
- $\neg x \leq \neg y \rightarrow x \leq y \rightarrow \sim x \leq \sim y$

Medieval Tree of Porphyry
Partitioned of the Genus by Privative Negation

Lasting Influence of Neoplatonic Hyper and Privative Negation on Classification and Theology (the *Via Negativa*)

- Pseudo Dionysius, *Divine Names, Mystical Theology,* 5th Century
- John Scottus Eriugena, 9th Century
- Bernard of Claivaux, 14th Century
- Peter Ramus, 16th Century
- *Port Royal Logic,* 17th Century
- Leibniz, *Theodicy*, 17th Century