Can there be an Inverted Square?

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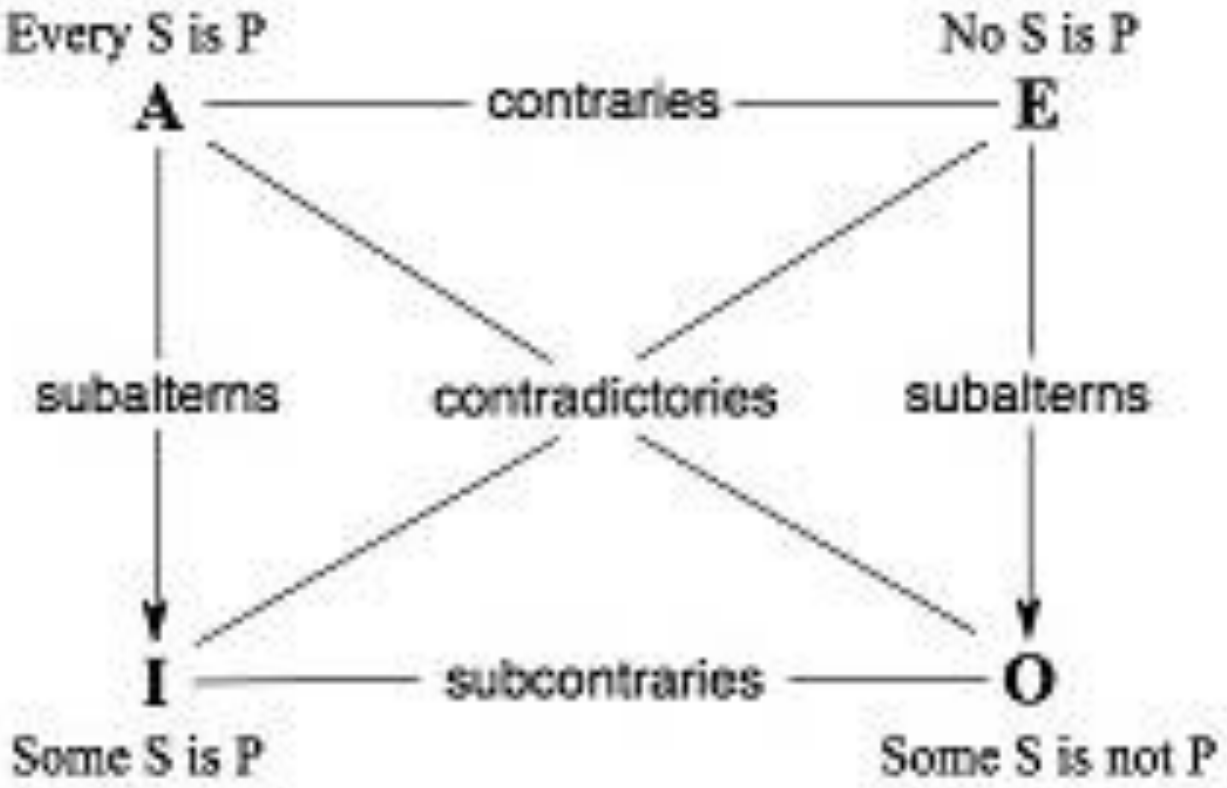
The beginning …

- In the nineteenth-twentieth century, logicians argued that certain relations between propositions as expressed in the traditional square, do not hold true as per the new developments taking place in logic. This paved way for a revised square. Modern logicians modified the traditional square with the relations of contrary, sub-contrary and sub-alternation falling apart, while contradiction remained intact in the modern square.

Abstract

- In this paper, we briefly compare and contrast these squares to demonstrate that the relationships of contrary, sub-contrary and sub-alternation actually holds in the latter square as well, along with contradiction. An upturned square might be possible if we draw the inverted square, which has skipped logicians' attention over the years.

Traditional Square of Opposition



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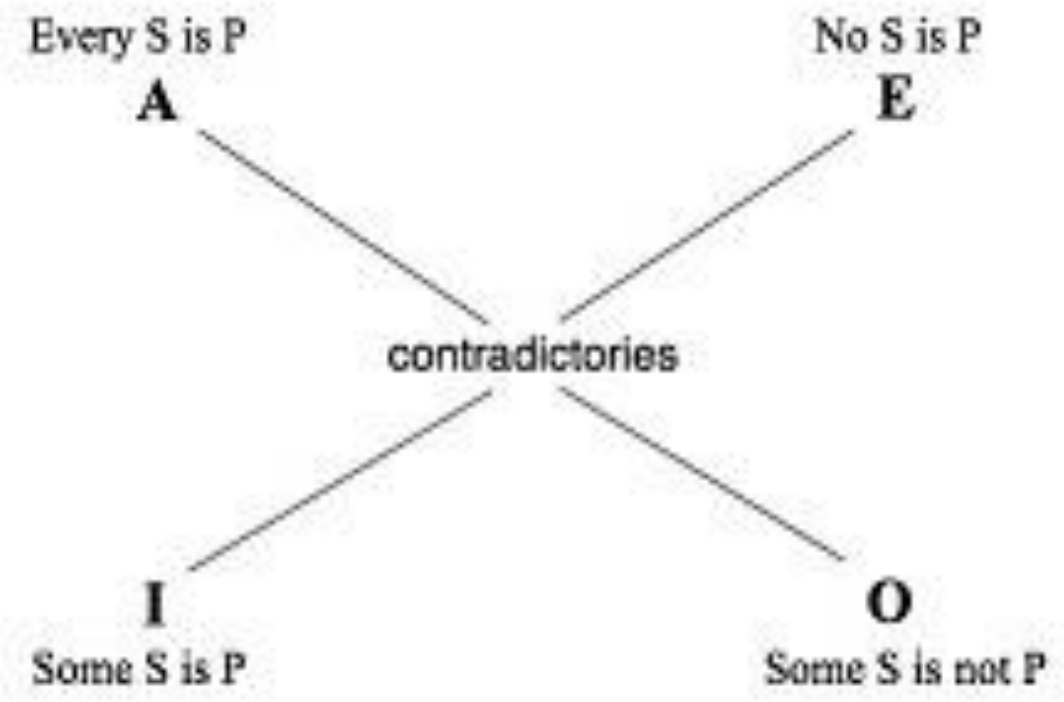
Revised Square of Opposition



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`Every S is P‘ represented as $(\forall x)[Sx \rightarrow Px]$

`No S is P‘ represented as $(\forall x)[Sx \rightarrow \sim Px]$

`Some S is P‘ represented as $(\exists x)[Sx \wedge Px]$

`Some S is not P‘ represented as $(\exists x)[Sx \wedge \sim Px]$

An argument from Copi

- Now suppose, if the subject class is empty then Sx will be false. If Sx is false, then A and E will be true because a false proposition implies any proposition whatsoever. However, their subaltern counterparts, namely, I and O will be false, since a conjunction is false, if at least one of its conjuncts is false. Thus, we obtain A and E as true whereas I and O as false.

Parsons' observations

In the Traditional Square, the particular negative proposition O is expressed as `Some S is not P'. This is a diachronic and lingual error, attributed to Boethius, who used `Not every S is P' and `Some S is not P' synonymously, while translating Greek to Latin. If we accept the above equivalence then `Every S is P' which can be vacuously true leads to `Some S is not P' true as well, after applying subalternation – contradiction - subalternation, respectively.

Parsons' observations … contd.

- In other words, if the subject term is empty then `Every S is P' is vacuously true, but its subaltern `Some S is P' will be false since S is empty. Now, if `Some S is P' is false then its contradictory `No S is P' is going to be true. Again, if `No S is P' is true then its subaltern `Some S is not P' has to be true. This defies the relation of contradiction.

Parsons' observations … contd.

- Parsons offers, at least a couple of defense to the above anomaly, which we found important here. First, that a universal proposition is vacuously true if its subject term is empty is a natural language nuance [of English language] which is not endorsed by many logicians. Second, a particular negative proposition needs to be symbolized as the conjunction of `Some S is S' and `Some S is not P', which will be false, if there are no Ss.

Read’s observation

- Read, while criticizing Łukasiewicz, is correct in pointing out that Aristotle commonly (though not invariably) expresses the O proposition as `Not every S is P' (or as he usually puts it: `P does not belong to every S'), and he clearly treats `P does not belong to every S' as equivalent to `P does not belong to some S'. Moreover, Aristotle places no requirement that the terms be non-empty. Existential commitment goes with quality, not quantity, thus satisfying all the demands of the [Traditional] Square of Opposition.

Corkum, Strawson and Wu

- Recently, Corkum has suggested that the existential import of universal affirmations and the semantic profile of predications with empty terms follow from mereological truth conditions. For example, `Socrates is pale' is true just in case `Socrates' is a part of the mereological sum of pale things.
- Strawson defended the Traditional Square by suggesting that the proposition whose subject term is empty is neither true nor false.
- Wu has opined that the problem [of existential import] lies in the gap between logic as pure abstraction and as a method applied to existence or human experience.

Uchenko and Northrop

- A discussion between Uchenko and Northrop, who are at loggerheads with each other, represented two powerful intuitions. Uchenko believed that the question of internal consistency is relative to information at hand. Therefore, the notion of correct and incorrect remains, system dependent.
- On the other hand, for Northrop, consistency is a consequence of certain formal principles, which cannot vary from system to system. Thus, two separate interpretations cannot be consistent together.

The discussion continues …

- A forthcoming article entitled, Incommensurability and Inapplicability of the Squares of Opposition" (in J.-Y. Beziau and I. Vandoulakis (eds), "The exoteric square of opposition“ to be published in the SUL series http://logica-universalis.org/sul Birkhäuser, Basel, 2020) discusses this point in detail from a paradigmatic point of view.

The Inverted Square?

a. Contrary relation does not hold in the modern square.

b. Sub-contrary relation does not hold in the modern square.

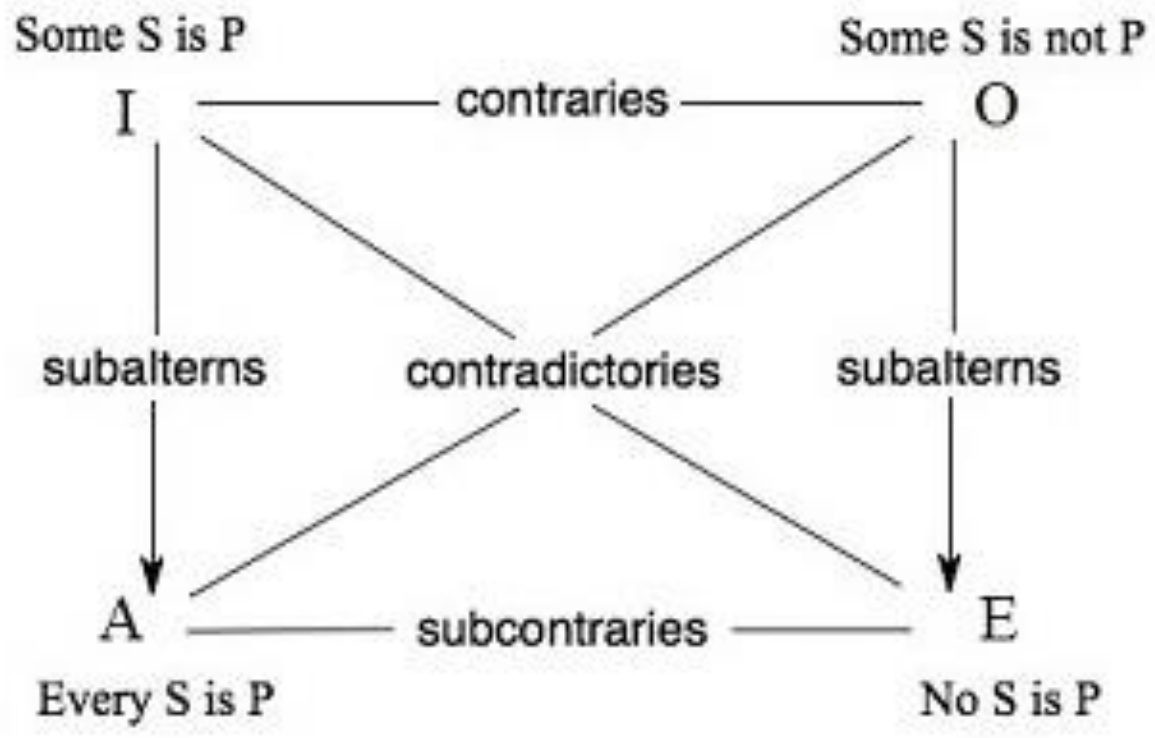
c. Sub-alternation relation does not hold in the modern square.

An incorrect truth table

Table 1. Truth Table of **A**, **E**, **I** and **O** Propositions

Sx	Px	$(\forall x)[Sx \rightarrow Px]$	$(\forall x)[Sx \rightarrow \neg Px]$	$(\exists x)[Sx \wedge Px]$	$(\exists x)[Sx \wedge \neg Px]$
T	T	T	F	T	F
T	F	F	T	F	T
F	T	T	T	F	F
F	F	T	T	F	F

The Inverted Square … continues …



Concluding Remarks

- These symbolic interpretations need further justifications or add-ons to reinstate the Traditional Square or we end up with a cross of opposition.
- There is a widespread belief that Aristotle's logic is not equipped to deal with empty terms, despite this belief being erroneous and baseless.
- Given the above state of affairs, it is unlikely that there can ever be a solution to this stalemate.

- Subalternation and existence presuppositions in an unconventionally formalized canonical square of opposition by Dale Jacquette.

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