

Andrzej Grzegorzcyk, a logician par excellence

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Abstract A short biography of Andrzej Grzegorzcyk (1922-2014) is presented, listing his main accomplishments in logic, mentioning his philosophical views, followed by a list of all his books and a selection of main papers.¹

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Mathematics Subject Classification 01A70

1 Life

Andrzej Grzegorzcyk was born in Warsaw on August 22, 1922, as the only son of Piotr, a historian of Polish literature, and Zofia who was a physician. He obtained his high school certificate in 1940, already during World War II. He studied physics and philosophy at the clandestine Polish university in Warsaw. He was first attracted to logic before the war when he heard on the radio a lecture of Jan Łukasiewicz on Stoic logic.

After the war Grzegorzcyk lived in Cracow, where he obtained MA in philosophy, and in 1946 he returned to Warsaw, where he became an assistant to Władysław Tatarkiewicz and a secretary of *Przegląd Filozoficzny* (Philosophical Review). He began an intensive study of logic and the foundations of mathematics, in part because this was the safest area of philosophy during the Communist rule. In 1950, he obtained PhD at the University of Warsaw. His dissertation, *On Topological Spaces in Topologies without Points*, was written under the supervision of Andrzej Mostowski. He was admitted to the Institute of Mathematics of the Polish Academy of Sciences, where after Habilitation he became a “docent” in 1953, an associate professor in 1961, and a full professor in 1972. Until 1968, he also lectured at the University of Warsaw. In 1974, Grzegorzcyk moved to the Institute of Philosophy of the Polish Academy of Sciences, and became the head of its Ethics Group in 1982. He retired in 1990. From 1999 to 2003 he served as the chairman of the Committee of Philosophy of the Polish Academy of Sciences. It is worth mentioning that after a series of lectures at the University of Amsterdam in 1965, he was proposed a permanent position there, but he refused since he did not want to leave Poland.

Often involved in organizing scholarly activities, he headed the 1973 Logical Semester that inaugurated the International Mathematical Center (the Banach Center) of the Polish Academy of Sciences. At that time it provided a rare occasion for extensive contacts of mathematicians from the Soviet bloc with those from the West. Cooperation with Russia and Ukraine was important for Grzegorzcyk, particularly after 1990. In 1995-1997 he led a special project, “One Hundred Years of the Lvov-Warsaw School”, which culminated in conferences in Lvov and Warsaw. Grzegorzcyk participated in many other conferences in logic and philosophy. He served as an assessor in the Executive Committee of the International Union of History and Philosophy of Science, the Division of Logic, Methodology and Philosophy of

¹ This paper is based on the biographies listed in Part III of the References section.

Science, and in 1979 he was elected to the Institut International de Philosophie. He received two honorary doctorates: from University of Clermont-Ferrand (2010) and from Jagiellonian University in Kraków (2013). He died in 2014.

In 1953 Andrzej Grzegorzczak married Renata Majewska, who would become a professor of Polish language and literature at the University of Warsaw. They have two children and six grandchildren.

In addition to his scholarly activities, Grzegorzczak was also a writer and was involved in public activities. During the Communist period, he signed some petitions protesting against limitations of freedom. He had life-long interest in the ideology of non-violence, a method of fighting without violence. Quite early he showed serious interest in environmental issues: in the 1970s, he popularized the warnings of the Club of Rome.

Grzegorzczak was a devout Roman Catholic, but he was a very independent Christian and was often critical of the official Church policies. He was also open to other faiths and churches, especially the Russian Orthodox Christianity. What is even more important, he was on good terms with intellectuals of various ideological convictions, religious and atheist, pro-Communist and anti-Communist.

2 Logical Accomplishments

Grzegorzczak's best known papers belong to logic and the foundations of mathematics. One of his early papers, the interpretation of Leśniewski's ontology as Boolean algebra without \emptyset (see P.5), suggested, according to him, that Leśniewski's calculus of names added nothing interesting to logic.

In the widely quoted 1953 paper *Some Classes of Recursive Functions* ([2]), he described and investigated classes of recursive functions obtainable by superposition, restricted recursion and the operation of restricted minimum (f is defined from g and h by: $f(n) =$ the smallest $x < h(n)$ such that $g(n,x)=0$) from some initial functions containing addition, in the next step multiplication, then raising to power, then superpowering, etc. This leads to a subrecursive hierarchy, that is, the so-called Grzegorzczak hierarchy. It forms a strictly increasing infinite sequence of classes of functions such that its union is equal to the class of primitive recursive functions. The third class of the Grzegorzczak hierarchy is identical with the class of elementary functions, definable as the smallest class of functions containing addition and subtraction, and closed with respect to superposition as well as restricted summation and restricted multiplication.

Grzegorzczak continued the investigations in computable analysis that had been initiated by Banach and Mazur (see P.4, 6, 7, 9). In particular, he co-edited (with Rasiowa) Mazur's posthumously published book *Computable Analysis*. Grzegorzczak also gave various definitions of computable real numbers. In this way he wanted to apply the concept of effectiveness used in the arithmetic of natural numbers in mathematical analysis. He also studied (see P.16) computable functionals of higher types.

In 1958 Grzegorzczak co-authored (with Mostowski and Ryll-Nardzewski) a fundamental paper "The classical and the omega-complete arithmetic" (P.10, also P.12) about second-order arithmetic, which treats it as formalized in first-order logic, but makes it possible to speak about natural numbers as well as their sets. Due to the introduction of the omega-rule, the Π_1^1 -relations are representable in it.

Working on computability, decidability and undecidability, Grzegorzczak considered various proofs of undecidability using recursively enumerable sets which are not recursive (see P.8),

and proved that elementary topological algebra, that is, Boolean algebra with closure, is undecidable, because arithmetic is interpretable in it (see P.3). He also showed (see P.14) that the calculus of combinators has no recursive model. His most recent contribution, “Undecidability without arithmetization” (2004, see P.26), deals with the theory of concatenation of texts (conceived as a primitive notion), introduced by Tarski in the 1920s. The theory of concatenation was shown by Grzegorzczuk to be undecidable; therefore, it and can replace arithmetization in metamathematics. Computability is replaced here by the effective recognizability of texts. The arithmetical relativization of quantifiers can be replaced by the relativization of expressions to subexpressions of a given expression.

Grzegorzczuk, like other Polish logicians and mathematicians, for example, Tarski, Mostowski, Rasiowa, Sikorski, was interested in intuitionistic logic and constructive mathematics. Whereas in his work on constructivism he looked at constructive mathematics from the point of view of classical methods, he believed that intuitionists may “perhaps penetrate more deeply into the meaning of logical concepts than classical logic does.” Grzegorzczuk’s semantics for intuitionistic logic introduced in “A philosophically plausible formal interpretation of intuitionistic logic” (1964) (see P.15, 17) is similar to the much better known construction by Kripke. It is based on the phrase “in my inquiry, I am forced to assert the sentence A at the moment t ,” and the notion of strong assertion, that is, an assertion that may not be abandoned at a later moment.

A modal interpretation of Grzegorzczuk’s semantics for intuitionism leads to the system named by Boolos S4.Grz, that is, the modal system S4 plus the formula

$$\Box(\Box(A \Rightarrow \Box A) \Rightarrow A) \Rightarrow A,$$

called Grzegorzczuk’s axiom. This system has applications in the provability logic.

Grzegorzczuk considered logic as belonging to the foundations of mathematics. In this field he worked according to Tarski’s program: every formal method is admissible in foundational research, independently of whether it is finitist, constructive or completely infinitistic.

3 Views

Among his ten philosophical books there is a study *Logic – a Human Affair* (1997), and (all in Polish): *Schemata and the Human Being* (1963), *Philosophy in the Time of Challenge* (1979), *Ethics in Internal Experience* (1989), *Life as a Challenge. Introduction to Rationalistic Philosophy* (1993), *Europe, Discovering the Sense of Existence* (2001), as well as a book of short literary forms *Moral Stories* (1986). He also published over sixty papers on mathematical and philosophical logic and over a hundred articles on ethics, religion, social issues.

Grzegorzczuk approved Łukasiewicz’s statement that logic is the morality of speech and thought. Logic was conceived by Grzegorzczuk in a broad sense as including semiotics and the methodology of science. According to him, logic in this sense constitutes a basic ingredient of European rationalism (see [18]). Since his youth he was convinced that everything can be expressed precisely and logically. He believed that European rationalism brought an exceptional intellectual and cultural success, and therefore must be protected. Teaching logic and proliferating its standards is a way.

For Grzegorzczuk, logic is a human affair. He maintained that semantic relations are always of someone and for someone. This leads to the acceptance of psychologism in logic. He also claimed that anti-psychologism inevitably leads to idealism. Grzegorzczuk argued that paradoxes should not be interpreted as showing that our language is inconsistent, but rather

that our concepts and conceptual systems are limited. For example, the Grelling antinomy shows that there exists a set of expressions which cannot be correctly named (see P.22) and the Liar paradox demonstrates the existence of problems about which nobody can think consistently, sincerely, and in a fully conscious way. In this approach, semantic antinomies appear as laws of thinking (see [17]).

Grzegorzcyk also studied axiomatic geometry based on the concept of solid (see P.13). He continued some works by Tarski, but he also motivated his work by referring to reism, a view formulated by Tadeusz Kotarbiński who claimed that individual spatio-temporal concrete things are the only elements of the furniture of the world. Grzegorzcyk was acquainted with reism by Henryk Hiż, his clandestine teacher of logic. Grzegorzcyk was always strongly attracted to this approach (see P. 1, 2, 19, 24), and believed that reism is in principle a good ontology for natural science. On the other hand, he clearly saw difficulties and limitations of reism in mathematics, particularly in set theory since it makes talking about infinite sets problematic.

Grzegorzcyk was a Christian and believed that Christianity represents the values of European rationalism. According to him, the history of Christianity shows the sense of the world by revealing the realm of the sacred and transcendence. He also admitted the presence of the highest values in other religions or ideologies. Humanity is not reducible to biological facts. European rationalism is open to moral values. Jesus's testimony represents the highest pattern of morality. At the same time, Grzegorzcyk was ready to (verbally) accept the need for some use of force in order to promote rational standards.

Grzegorzcyk was strongly attracted to the attitude of radical non-violence, as represented by Mahatma Gandhi or Martin Luther King. He helped organize symposia on this issue. He was uncompromising in recommending compromise in all conflicts. We should talk with everyone, he claimed, also with a terrorist. He was also one of the first people in Poland accepting ecological challenges; he proposed limitations on consumption and opposed waste, although such appeals sounded highly abstract in Poland of the 1970s.

He considered moral issues and social challenges to be so important in the contemporary world that he criticized as immoral the focus, so common among the brightest minds, on abstract intellectual questions which can have no impact on social problems. His move from the Institute of Mathematics to the Institute of Philosophy and Sociology was partly motivated by this frustration. He felt he participated in the malaise. After all, he avowed that for him mathematical problems in formal logic were an "obsession, addiction, narcotic."

He believed that we, humans, need solidarity of all people, independently of where they live. He even appealed to the UN in the 1970s proposing to establish a law that everybody is entitled to help every person who is in a worse situation, wherever the other person lives. In particular, richer people or societies should help poorer ones, the stronger should protect the less powerful, the better educated should do something for the less educated, etc.

Grzegorzcyk's religiosity and his understanding of religion provided an extension or correction to his views on the supremacy of logic in all reasoning, including the moral debates. In 1974 he wrote a poem in which it was stated that logic is powerless in dealing with religious matters: "I do not refute those who say You don't exist. I agree with the friends who say that Your concept is contradictory... only those have a pure concept of God who don't have it at all. ..."

4 Influence

Grzegorzcyk will remain known due to his logical achievements: the Grzegorzcyk hierarchy, results about undecidability, second-order arithmetic, the S4Grz system of modal logic, semantics for intuitionistic logic, his concatenation theory. Also his course in mathematical logic *An Outline of Mathematical Logic* (in Polish 1961, in English 1974, see [6], [8]) was for a long time the only detailed textbook in Polish in which logical calculus, model theory and recursion theory were presented in a balanced fashion, and influenced many students of mathematics and philosophy in Poland. His textbook on the foundations of arithmetic *An Outline of Theoretical Arithmetic* (1971, in Polish, see [7]), in which he simultaneously developed arithmetic and exposed its logical basis, was much less influential. He popularized recursive functions and the problems of computability and decidability not only in Poland but also in France (see [3], [4]).

Grzegorzcyk was productive but he was always working alone as it was never easy to work with him. He established no school, and hardly had students. He supervised only two PhD dissertations, one in logic and one in ethics. The one in logic was mine, but as a matter of fact I received little from him; I owed my logical education primarily to Andrzej Mostowski.

Grzegorzcyk was not a member of any ideological group, and in no circle was he considered fully "one of our guys". At the same time he was widely respected for being absolutely honest and sincere. Whereas he was recognized as an important logician, Grzegorzcyk's philosophical and axiological views did not become important or influential. His other works were rarely noticed. There was a notable exception: in 1987 he obtained a literary prize for the book "Moral stories".

In addition to particular formal achievements two general main messages are expressed in Grzegorzcyk's work: first, the utmost importance of logical thinking, second, the all-human solidarity. I believe that for all who knew him he can easily remain the best available personification of logic, or the individual embodying the ideal type of a logician. This is a rather ambivalent tribute, even among logicians.

5 References

In Part I all Grzegorzcyk's books are listed, and in Part II a selection of his articles. A fairly comprehensive list is available in the paper III.1 mentioned below.

I. Books

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