A view of revival of Mathematical Logic in Warsaw, 1945 – 1975

Victor W. Marek

To the memories of my teachers: Andrzej Mostowski, Helena Rasiowa, and Andrzej Grzegorczyk

Abstract. This is a (biased) account of the revival of Foundations of Mathematical Logic, and more generally Foundations of Mathematics after WWII. The perspective is limited to years 1945-1975. This coincided with the activities of Professor Andrzej Mostowski, the principal leader of the foundational research in Poland during the reported time. Moreover, in this text, we focus on the development of Foundations in Warsaw, which misses an important aspect of the foundational research in Poland of the time, specifically in Wrocław. Even the description of the Warsaw efforts is incomplete, not covering Foundations of Geometry. We focus on activities of three main researchers: Andrzej Mostowski, Helena Rasiowa and Andrzej Grzegorczyk.

Mathematics Subject Classification (2000). Primary 01A60, 03-03.

Keywords. Logic in Warsaw, Andrzej Mostowski, Helena Rasiowa, Andrzej Grzegorczyk.

1. Introduction

World War II separates significant achievements of mathematical community of Poland during the so-called "Second Polish Republic" (1918 - 1939) and very different "Polish Peoples Republic" that was created after liberating Poland from German occupation. The war resulted in significant territorial changes. In effect, Poland was moved some 200 miles to the west, loosing significant part of its Eastern part and gaining territories to the West of the pre-war borders that had previously been German. Poland lost two significant academic centers: Lwów, (now Lviv in

The author is grateful to Andrzej Jankowski, Stanisław Krajewski, Marcin Mostowski and Andrzej Skowron for their corrections and other helpful suggestions.

Ukraine) and Wilno (now Vilnius in Lithuania). Instead, Poland gained important centers of science: Wrocław (previously German Breslau) as well as Gdańsk (previously a free city). Significant part of Prussia became part of Polish territory.

Science in Poland, and in particular Mathematics (thus also Mathematical Logic) suffered greatly as a result of German occupation and, more generally, World War II. The losses were both due to extermination of intellectuals by Germans (especially of those with Jewish roots) and emigration. Logicians such as Adolf Lindenbaum, Mordechaj Wajsberg, Mojzesz Presburger, and others were killed by Germans. Several important logicians, in particular Alfred Tarski, Jan Łukasiewicz, Bolesław Sobociński and others left Poland and emigrated as the result of the war and continued their work in foreign institutions.

There was a conscious effort to re-create Jan Kazimierz University of Lwów in Wrocław, and to some extent, Lwów mathematics, with Hugo Steinhaus continuing Lwów traditions. Likely the result of WWII, the death of Stefan Banach in 1945, in a relatively early age of 53, deprived Poland of one of its great mathematicians open to many foundational challenges.

2. The beginnings

It is very hard to imagine the amount of destruction both of Warsaw and Wrocław as the result of WWII. The western part of Warsaw (where the Warsaw University was, and is now) was destroyed in 95%. This author, born in the second half of WWII, remembers well the sea of ruins in parts of Warsaw. This was the result of a systematic destruction of that part of Warsaw by Germans after the fall of Warsaw Uprising (August – October 1944.) Wrocław, converted by Nazis into a fortress, was similarly destroyed. Both places were, essentially, human deserts.

During the WWII, there were several underground Universities in Warsaw, functioning in spite of prohibition by the occupiers of both secondary and university education. At a significant risk, the principal Warsaw mathematicians (Sierpiński, Kuratowski, and others) were conducting lectures and doing research. All of them left Warsaw with the rest of the population when Germans started their destruction of the city.

Among the other mathematicians active in the underground Warsaw University was Andrzej Mostowski, a student of Lindenbaum, Tarski, Gödel and Kuratowski. Mostowski conducted an active research during WWII (a story of his "black notebook" containing the results of his research and the alternative he faced as he was leaving Warsaw in 1944 is told in several publications devoted to his life). Mostowski, along with the rest of population of Warsaw had to leave the town. Eventually, he found his way to Kraków. In 1945, Mostowski received his *habilitation* degree at Jagiellonian University. Mostowski returned to Warsaw in 1946, and was associated with the revived Warsaw University till the end of his life in 1975.

The faculty of Warsaw University with interests in widely understood Logic and Foundations, Kazimierz Kuratowski and Waclaw Sierpiński, while still active, had principal interests in a variety of areas, with Kuratowski mainly interested in Topology, and Sierpiński in Number Theory and Combinatorial Set Theory. Both of them had residual interest in Foundations, but it was not the principal area of their research. In this situation, the fundamental role in rebuilding foundational research in Poland fell on Mostowski, who, eventually, educated a large group of students who helped him in returning Warsaw to one of the prime centers in foundational research. Already in 1946, both Helena Rasiowa and Wanda Szmielew started their research in logic (both studied in the underground Warsaw University during German occupation). Eventually, both Professor Rasiowa and Professor Szmielew developed interests in areas not directly related to Professor Mostowski research (with Rasiowa mainly interested in Algebraic Foundations of Logic, and Szmielew working in Foundations of Geometry.) Both Rasiowa and Szmielew educated groups of logicians following the directions of research outlined above. Additionally, Professor Rasiowa interests were later influenced by Computer Science applications of Logic. The results of Professor Szmielew initially developed during German occupation and then continued during her stay at the University of California, Berkeley, 1949-1950, (decidability of the theory of Abelian groups) were instantly recognized as a major result. Even earlier, 1948-1949, Professor Mostowski spent a year as a visitor at the Institute of Advanced Studies, in Princeton, NY. Mostowski's relation with Gödel (since Mostowski's extended visit in Vienna in 1936) played an important role in Mostowski's research in several areas of Foundations, specifically in the metamathematics of set theory and related formalisms.

3. Andrzej Mostowski and his students

The most amazing aspect of Mostowski's activities during the first 15 years after WWII is the breadth of his interests as reflected in his publications. He found significant results in Recursion Theory, Set Theory, Proof Theory, and Model Theory. This needs to be related to the fact that the notebooks that he wrote during WWII were lost (Germans allowed inhabitants of Warsaw expelled from the city to take one kilogram of items, and he (as stated to this author) chose a kilogram of bread). Reconstruction of the results and their proofs took time, and not all was remembered. Still, results on independence of various set-theoretic statements, results on existence of models of arithmetical theories, results on the arithmetical hierarchy, on proof theory of intuitionistic logic and results on the incompleteness of Peano arithmetic and its extensions (culminating in the monograph with R. Robinson and A. Tarski) all followed in quick succession. The handbook on Mathematical Logic (in Polish, 1948) was a powerful tool in educating generations of Polish logicians [1]. Almost every year in the 1950ies Mostowski was publishing

[1] I believe that one of the articles in the present volume discusses this monograph.

work that was widely read, commented, and even more importantly further expanded by mathematicians all over the world. It is enough to mention his work on the direct product of theories (significantly expanded by Solomon Feferman and Robert Vaught), generalized quantifiers (which pioneered the area of significant importance both in mathematical logic and (much later) computer science, model theory of second order arithmetic (together with Andrzej Grzegorczyk and Czesław Ryll-Nardzewski), and one of most important tools of model theory - existence of models with indiscernibles (joint work with Andrzej Ehrenfeucht). This created the opportunity for many individuals to study with Professor Mostowski[2].

It is safe to say that by the end of 1950ies, Warsaw, and more generally, Poland, returned to its place as one of most important centers of research in the Foundations of Mathematics. Moreover, by that time, a cadre of researchers both within direct circle of Mostowski, as well as circles of his former students, such as Andrzej Grzegorczyk, Helena Rasiowa and Wanda Szmielew was active in Warsaw. Each of these individuals (and, of course, Professor Mostowski himself) directed a seminar at Warsaw University. For that reason, the access to the knowledge in the areas of Foundations was easy both for students and faculty.

The unusually wide interests of Mostowski continued into 1960ies. The breakthrough in the area of independence proofs in Set Theory due to Paul J. Cohen using the technique of forcing influenced Mostowski and his group of collaborators. Mostowski devoted to this topic a monograph that presented Cohen arguments in a new perspective. Even more importantly, in a series of lectures at Helsinki (subsequently published as a monograph), Mostowski presented a unified perspective of Foundations of Mathematics of the middle of 1960ies. It established him as one of the leaders and visionaries of the area.

Continuing exploration of other areas of foundations, models of second order arithmetic, impredicative theories of classes, abstract model theory, weak systems of arithmetic and other current areas of research, Mostowski constantly expanded the perspective of research in Warsaw. A number of scientific meetings in Warsaw, culminating with the Foundations of Mathematics Semester at the Banach Center in 1973, firmly established Warsaw as a place worth visiting. During 1960ies and first half of 1970, there was a constant stream of visiting logicians benefiting of Mostowski's hospitality and the cutting edge of foundational research in Warsaw.

Professor Mostowski's death in 1975, at age 62, was a significant blow to foundational work in Warsaw, and more generally, in Poland. Additionally, later on, important political disturbances, including martial law, resulted in further losses in the foundational research in Warsaw.

A most comprehensive and reasoned bibliography of Professor Andrzej Mostowski is provided by Jan Zygmunt [12] and supersedes earlier efforts by students of Mostowski. The bibliography lists over 110 papers and their translations including papers published posthumously) as well as over 15 monographs and handbooks.

[2]The author of the present article was Mostowski's Ph.D. student in 1964-1968.

Together with a large number of reviews of papers, this bibliography presents the enormous body of highest-class research and other contributions to Foundations.

At present, Professor Mostowski has 18 direct descendants in the Mathematical Genealogy Database as well as 300 indirect descendants. Since some Mostowski's students and their descendants moved to Computer Science and Philosophy, the number of indirect descendants is, clearly, bigger.

4. Helena Rasiowa and her group

Professor Helena Rasiowa started her Mathematics studies in 1938. The World War II, with its consequences for Polish citizens (with dual occupation at first, and then suffering under German occupation) limited opportunities for individuals interested in higher education to "underground University". Rasiowa was studying in that institution, and wrote her M.Sc. thesis under the direction of Łukasiewicz and Sobociński. She described to her younger collaborators (this author included) the horrible experience of being buried under ruins of a house bombed during the Warsaw Uprising of 1944. As the rest of the population of Warsaw she was expelled of the city, and survived as a secondary school teacher. Andrzej Mostowski brought her back to the Warsaw University where she started her Ph.D. studies under his guidance. Her M.Sc. thesis was reconstructed (recall Mostowski's experience mentioned above). Following Mostowski's work on algebraic methods of logic (the area 100 years old at the time, already), Rasiowa developed algebraic techniques for modal and intuitionistic logics. Soon, jointly with Roman Sikorski, she developed a comprehensive theory of algebraic methods in logic. Several papers in this area resulted in habilitation at the Mathematical Institute of the Academy of Sciences. The essence of that theory consisted in expanding the notion of semantics to one where values form a Boolean Algebra (or Heyting algebra, in intuitionistic case). A fundamental result of this are, often called Rasiowa-Sikorski Lemma proves the existence of filters closed under denumerable family of meets. This result provides an alternative means to prove completeness theorem for predicate calculus and is Baire Theorem in disguise. It also explains why Cohen's technique of forcing works (although Rasiowa and Sikorski did not take an opportunity to provide this, maybe the most spectacular, application of their techniques in their famous monograph "Mathematics of Metamathematics").

The work on the algebraic methods in logic continued throughout the entire further life of Rasiowa. She created a large group of collaborators who worked on algebraic methods in a variety of logics, including logics that stemmed of application of formal methods in Computer Science.

Professor Rasiowa was very active in the science administration, both at Warsaw University (she served as a Dean of Mathematics and Physics, later Mathematics, Mechanics and Computer Science for many years) and international scientific

institutions. She helped to establish *Studia Logica* as a leading journal on the borderline of Mathematics and Philosophy, and *Fundamenta Informaticae* (together with Zdzisław Pawlak).

Beginning in 1960s Professor Rasiowa promoted foundational issues in Computer Science; her contributions to the logic of programs (algorithmic logic) formed a widely-read text published by the Institute of Computer Science of Polish Academy of Sciences.

Professor Rasiowa supervised 19 PhD dissertations. At present she has 12 direct descendants in the Mathematical Genealogy Database as well as 93 indirect descendants. Since some (in fact most) of her students and their descendants moved to Computer Science, the number of indirect descendants is, clearly, bigger.

The paper by Bartol, Orłowska and Skowron includes the full list of Rasiowa's work.

5. Andrzej Grzegorczyk

Of the four main leaders of mathematical logic, and more generally, foundations of mathematics in post 1945 Poland, the youngest researcher was Professor Andrzej Grzegorczyk (1922-2014).

Professor Grzegorczyk studied (like Wanda Szmielew and Helena Rasiowa) at underground universities in Warsaw where he studied chemistry, physics and philosophy. He completed his studies in Cracow, immediately after the war; his MA thesis generalized Leśniewski's system to higher types. His interests at the time were motivated by philosophical questions - and involvement in widely understood philosophy continued throughout his entire scientific career. After WWII, Grzegorczyk turned to mathematical logic, with his Ph.D. (written under the direction of Andrzej Mostowski) devoted to formal topology. Several important contributions to Foundations of Mathematics followed. Of these, the hierarchy of primitive-recursive functions is commonly called *Grzegorczyk hierarchy*. The paper on the hierarchy can be treated as one of the first research papers on computational complexity of recursive functions. In 2003 Grzegorczyk proposed an new proof for Gödel's undecidability result. The proof omits arithmetization. Grzegorczyk's proof is much simpler than the original Gödel's proof. Other important contributions involved modal logic (with the logic commonly called *Grz*, Leśniewski's mereology and λ-calculus). Joint work with Mostowski and Ryll-Nardzewski on ω-models of second-order arithmetic established fundamental properties of these structures, creating foundations for significant body of work in Warsaw.

Professor Grzegorczyk, like many philosophically-oriented logicians was concerned with the issues of syntax and definability of fundamental notions. This, to some extent, resulted in tensions between Professor Grzegorczyk and other researchers in foundations, esp. mathematicians. The point was that mathematicians proved correctly results over the millenia, and these results, and more importantly,

their proofs are still valid. This results in a certain attitude among mathematicians ("this is philosophy, not mathematics") which alienates some philosophical thinkers. Eventually, Professor Grzegorczyk left mathematics (at least formally) transferring to the Institute of Philosophy and Sociology of the Academy of Sciences.

Of the researchers of foundations of older generation, Professor Grzegorczyk was the most concerned with the questions of ethics. A rigid set of values and opinions, often going "against the grain" distinguished Grzegorczyk among his peers. During the "real socialism" he was willing to protest against injustices (this author benefited from Grzegorczyk's help and encouragement when subjected to repressions), but was willing to speak against popular opinions when he felt that these bring more harm than societal benefit. As a result, Grzegorczyk stood out among many, but "not with many".

The complex nature of the research of Professor Grzegorczyk (firmly placed on the intersection of Mathematical Logic and Philosophy) resulted in some controversies, esp. from the mathematical side and critiques of his fundamental contribution to Foundations of Mathematics, *Outline of Mathematical Logic*.

Mathematical Genealogy Projects lists one direct descendant of Professor Grzegorczyk and 6 indirect descendants. Since the unique descendant listed in that project is a Computer Scientist, the number of descendants is, likely, bigger. The influence of Professor Grzegorzyk on the foundational research in Philosophy was also significant.

More details and references to Grzegorczyk's detailed bibliography can be found in the paper by Krajewski in this volume.

6. Conclusions

The picture of revival of the Warsaw School of Foundations after the WWII that we present in this notes, raises a variety of issues. The first, and the most important one refers to the role of an individual in the scientific process. Certainly, the developments in Warsaw after 1945, were dominated by the contributions and the personality of Professor Andrzej Mostowski. The two other individuals treated in this text, Professor Helena Rasiowa and Professor Andrzej Grzegorczyk were able to contribute to the area of Foundations, but in the reported period, practically all foundational research in Warsaw (possibly with the exception of the work by Professor Wanda Szmielew group) was originally motivated by Professor Mostowski. He not only conducted his own research and motivated his collaborators, but also initiated research of both Professors Rasiowa and Grzegorczyk.

This raises the question of dependence of such scientific events (revival of a scientific school) on the individuals. Of course, it is difficult to hypothesise, but what would happen if Professor Mostowski did perish during the German occupation? Would the Warsaw school revive?

While one may wonder if such questions are meaningful, the many disasters during the second half of 20^{th} century and also more recent violent developments in various parts of the world raise the issue on the dependence of scientific "schools" on the availability and contributions of individuals. Maybe the events and developments of the Warsaw School of Foundations tells us something and suggest blueprint for revival in analogous situations.

Andrzej Mostowski

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Victor W. Marek
Department of Computer Science University of Kentucky, Lexington, KY 40506, USA
e-mail: marek@cs.uky.edu