Ontology of logic and mathematics in Lvov-Warsaw School

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Abstract The aim of the paper is to consider ontological views connected with mathematics and logic of main representatives of Lvov-Warsaw School of Philosophy. In particular views of the following scholars will be presented and discussed: Jan Łukasiewicz, Stanisław Leśniewski, Alfred Tarski, Tadeusz Kotarbiński and Kazimierz Ajdukiewicz. We shall consider also views of Andrzej Mostowski who belonged to the second generation of the school as well as of Leon Chwistek who was not directly the member of this group but whose conceptions are of interest.

Keywords Philosophy of logic, Philosophy of mathematics, Ontology, Platonism, Nominalism, Intuitionism

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The aim of the paper is to describe ontological views concerning logic and mathematics of representatives of Lvov-Warsaw School of Philosophy.[1] As will be shown there was in fact no common conception. The element unifying members of the school was not a particular philosophical doctrine but rather the method of practicing and developing philosophy. I. Dąmbska charactered it in the following way:

> There was no common doctrine, no uniform view of the world shared by Lvov philosophers. What formed the foundation of the spiritual community of those scholars was not the content of conceptions but rather the way, the method of doing philosophy as well as the common scientific language. Thanks to that feature members of this group could be both spiritualists and materialists, nominalists and realists, logicians and psychologists, philosophers of nature and theoreticians of art.[2] ([8], p. 17)

1 Jan Łukasiewicz − (Neo)Platonism

[1] On Lvov-Warsaw School in Philosophy see Woleński [40] and [41] as well as Murawski [37] and [38].

[2] Nie łączyła bowiem filozofów lwowskich jakaś wspólna doktryna, jakiś jednolity pogląd na świat. To, co stworzyło podstawę wspólnoty duchowej tych ludzi, to była nie treść nauki, tylko sposób, metoda filozofowania i wspólny język naukowy. Dlatego wyjść z tej szkoły mogli: spirytualiści i materialiści, nominaliści i realiści, logistycy i psychologowie, filozofowie przyrody i teoretycy sztuki.

Let us start by considering views of Jan Łukasiewicz (1879−1956). One should begin by stressing his anti-psychological attitude. Psychologism claims that the objects investigated by logic and mathematics exist as psychological beings and are got to know like other psychological facts. This approach was popular in the philosophy of logic and mathematics at the end of the 19th century. It was criticised in particular by Frege, Husserl and Meinong. In the paper "Logika a psychologia'' [Logic and psychology] (cf. [21]) Łukasiewicz formulated his arguments against psychologism. First he stated that laws of psychology are empirical and consequently only probable whereas laws of logic are certain. Laws of both those disciplines are also of different character: the laws of logic concern relations between the truth and falsity of judgements whereas the laws of psychology state the relations between psychological phenomena. Łukasiewicz concludes:

> Exposing the attitude of logic towards psychology can be to the advantage of both sciences. Logic will be cleared from the weeds of psychologism and empiricism, which choke its right development and the psychology of cognition will get rid of *a priori* traces which hid the light of the sincere splendour of its truths. Since one should remember that logic is an *a priori* science, like mathematics, whereas psychology, like any other natural science, is based and must be based on experience.[3]

Łukasiewicz stressed explicitly the apriorism of logic. In the paper "O twórczości w nauce'' [On creativity in science] (cf. [22]) he wrote:

> Logic is an *a priori* science. Its theorems are true by virtue of definitions and axioms flowing from reason and not from experience. This science is a domain of pure mental creativity. […] Logical and mathematical judgements are truths only in the world of ideal beings. We will never know whether some real objects correspond with these beings.
>
> *A priori constructions of the mind, being part of every synthesis, imbue the whole*

[3] Wyświetlenie stosunku logiki do psychologii przynieść może korzyści obu tym naukom. Logika oczyści się z chwastów psychologistycznych i empirystycznych, które tłumią jej prawidłowy rozwój, a psychologia poznania pozbędzie się naleciałości apriorycznych, spod których szczery blask jej prawd nie mógł jakoś dotąd zajaśnieć. Należy bowiem pamiętać, że logika jest nauką aprioryczną, tak jak matematyka, a psychologia, tak jak każda nauka przyrodnicza, opiera się i opierać się musi na doświadczeniu. ([21], p. 491)

science with an ideal and creative element.[4]

Łukasiewicz admitted that logic and mathematics have a nominalistic robe (see for example his paper "Logistyka a filozofia" [Logistics and philosophy], [24], p. 119) but simultaneously he saw some difficulties in the nominalistic approach. An individual can create only a finite number of inscriptions. Hence a set of inscriptions is finite what would mean that the set of theses of logic and mathematics would be finite as well but "on this basis it would be as difficult to practise logistics, especially metalogistics, as to build arithmetic on the assumption that the set of natural numbers is finite"[5] ([27], p. 224). It would also lead to make logic dependent on certain empirical facts, i.e., on the existence of inscriptions, which is difficult to accept.

According to Łukasiewicz the nominalism of logic and mathematics is virtual. Moreover, logic was developed without solving the problem of its nominalism. In his article "Logistyka a filozofia" [Logistics and philosophy] he wrote:

> We have so far been little worried by these difficulties, and this is the strangest point. It was so probably because, while we use nominalistic terminology, we are not true nominalists but incline toward some unanalysed conceptualism or even idealism[6] ([26], p. 224).

Łukasiewicz himself thought that the objects that logic investigated existed only beyond the sphere of inscriptions. He did not develop some alternative to nominalism – he just formulated his personal view. But his view resulted from his personal religious convictions – influenced by these convictions Łukasiewicz opted for the Neoplatonic interpretation of logic. In the paper "W obronie logistyki" [In defence of logic] he wrote:

In concluding these remarks I should like to outline an image which is connected with

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[4] Logika jest nauką aprioryczną. Twierdzenia jej są prawdziwe na mocy określeń i pewników płynących z rozumu, nie z doświadczenia. Nauka ta jest dziedziną czystej twórczości myślowej. [...] Sądy logiczne i matematyczne są prawdami jedynie w świecie bytów idealnych. Czy bytom tym odpowiadają jakieś przedmioty rzeczywiste, o tym zapewne nigdy się nie dowiemy.

Aprioryczne konstrukcje umysłu, wchodząc w skład każdej syntezy, przepajają całą naukę pierwiastkiem idealnym i twórczym. ([22], pp. 13−14)

[5] na takiej podstawie byłoby równie trudno uprawiać logistykę, a zwłaszcza metalogistykę, jak trudno byłoby zbudować arytmetykę na gruncie założenia, że zbiór liczb naturalnych jest skończony. ([23], p. 120)

[6] Mało dotychczas przejmowaliśmy się tymi trudnościami i to jest w tym wszystkim najdziwniejsze. Działo się to chyba dlatego, że używając terminologii nominalistycznej, nie jesteśmy naprawdę nominalistami, lecz hołdujemy jakiemuś nie zanalizowanemu konceptualizmowi czy nawet idealizmowi. ([23], p. 120)

> the most profound intuitions which I always experience in the face of logistic. That image will perhaps shed more light on the true background of that discipline, at least in my case, than all discursive description could. Now, whenever I work even on the least significant logistic problem, for instance, when I search for the shortest axiom of the implicational propositional calculus I always have the impression that I am facing a powerful, most coherent and most resistant structure. I sense that structure as if it were a concrete, tangible object, made of the hardest metal, a hundred times stronger than steel and concrete. I cannot change anything in it; I do not create anything of my own will, but by strenuous work I discover in it ever new details and arrive at unshakable and eternal truth. Where is and what is that ideal structure? A believer would say that it is in God and is His thought.[7] ([27], p. 249)

Łukasiewicz stressed that this was his personal view. He was of the opinion that logic is neither called nor allowed to solve the eternal philosophical debate concerning universals.

2 Stanisław Leśniewski − Nominalism

Nominalism mentioned above was the philosophical doctrine of another representative of Lvov-Warsaw School in Philosophy and, together with Łukasiewicz, the founder of Warsaw School of Logic, Stanisław Leśniewski (1886−1939). This doctrine had strong influence even on the contents as well as on the form of his logical constructions. His views Leśniewski called constructive nominalism.

Leśniewski treated language as a collection of concrete inscriptions and expressions of a language as finite sequences of signs. Two inscriptions of the same shape were treated by him as two separate, different inscriptions. In his opinion there only exist as many

[7] Chciałbym na zakończenie tych uwag nakreślić obraz związany z najgłębszymi intuicjami, jakie odczuwam zawsze wobec logistyki. Obraz ten rzuci może więcej światła na istotne podłoże, z jakiego przynajmniej u mnie wyrasta ta nauka niż wszelkie wywody dyskursywne. Otóż ilekroć zajmuję się najdrobniejszym nawet zagadnieniem logistycznym, szukając np. najkrótszego aksjomatu rachunku implikacyjnego, tylekroć mam wrażenie, że znajduję się wobec jakiejś potężnej, niesłychanie zwartej i niezmiernie odpornej konstrukcji. Konstrukcja ta działa na mnie jak jakiś konkretny dotykalny przedmiot, zrobiony z najtwardszego materiału, stokroć mocniejszego od betonu i stali. Nic w niej zmienić nie mogę, nic sam dowolnie nie tworzę, lecz w wytężonej pracy odkrywam w niej tylko coraz to nowe szczegóły, zdobywając prawdy niewzruszone i wieczne. Gdzie jest i czym jest ta idealna konstrukcja? Filozof wierzący powiedziałby, że jest w Bogu i jest myślą Jego. ([24], p. 165).

expressions as they have been written. One cannot speak of some potential existence of expressions. Consequently a given logical system contains only so many theorems as they have been written until a given moment, i.e., every logical system consists of only a finite number of theorems. Leśniewski did not allow the existence of any general objects, in particular of common properties of individual objects. Another consequence of Leśniewski's nominalism was the fact that two equivalent systems, for example the system of propositional calculus based on negation and implication and the system based on negation and disjunction as primitive connectives usually treated as two variants of the same logic should be treated now as two different systems. Leśniewski's systems are never something complete at a given moment.

Leśniewski connected the described view with the so-called intuitive formalism. According to it a language of logic − uniquely and completely codified − says always "something" and about "something". In the work "Grundzüge eines neuen System der Grundlagen der Mathematik" [18] he wrote:

> Having no predilection for various 'mathematical games' that consist in writing out according to one or another conventional rule various more or less picturesque formulae which need not be meaningful, or even − as some of the 'mathematical gamers' might prefer − which should necessarily be meaningless, I would not have taken the trouble to systematize and to often check quite scrupulously the directives of my system, had I not imputed to its theses a certain specific and completely determined sense, in virtue of which its axioms, definitions and final directives, have for me an irresistible intuitive validity.[8] ([19], p. 487)

In the work "O podstawach matematyki" [On the foundations of mathematics] (cf. [17]) one reads:

> They encouraged the disappearance of the feeling for the distinction between the

[8] Da ich keine Vorliebe für verschiedene «Mathematikspiele» habe, welche darin bestehen, dass man nach diesen oder jenen konventionellen Regeln verschiedene mehr oder minder malerische Formeln aufschreibt, die nicht notwendig sinnvoll zu sein brauchen oder auch sogar, wie es einige der «Mathematikspiele» lieber haben möchten, notwendig sinnlos sein sollen, - hätte ich mir nicht die Mühe der Systematisierung und der vielmaligen skrupulösen Kontrollierung der Direktiven meines Systems gegeben, wenn ich nicht in die Thesen dieses Systems einen gewissen ganz bestimmten, eben diesen und nicht einen anderen, Sinn legen würde, bei dem für mich die Axiome des Systems und die in den Direktiven zu diesem System kodifizierten Schluss- und Definitionsmethoden eine unwiderstehliche intuitive Geltung haben ([18], p. 78).

mathematical sciences, conceived as deductive theories, which serve to capture various realities of the world in the most exact laws possible, and such non-contradictory deductive systems, which indeed ensure the possibility of obtaining, on their basis, an abundance of ever new theorems, but which simultaneously distinguish themselves by the lack of any connection with reality of any intuitive, scientific value.[9] ([20], pp. 177−178)

Leśniewski treated formal systems as a means to transmit certain information about the world and as a way to express what is intuitively true. This may seem not to be fully in accordance with his nominalism and radical formalism. However he did not consider those views as contradictory. In fact in "Grundzüge eines neuen System der Grundlagen der Mathematik" [18] he wrote:

I see no contradiction, therefore, in saying that I advocate a rather radical 'formalism' in the construction of my system even though I am an obdurate 'intuitionist'. Having endeavoured to express some of my thoughts on various particular topics by representing them a s a series of propositions meaningful in various deductive theories, and to derive one proposition from others in a way that would harmonize with the way I finally considered intuitively binding […].[10] ([20], p. 487)

For Leśniewski logic was the description of most general features of being (the same − under the influence of Leśniewski − was claimed by Kotarbiński). Hence it plays the role of a general theory of objects. This view was in accordance with the fact that Warsaw School of Logic rejected the so-called analytic interpretation of logic, i.e., the thesis that logic and mathematics are the set of tautologies that do not say anything about the world. Logic and mathematics were thought to refer to the formal aspects of reality. Add that Leśniewski rejected also the conventionalism in the style of Poincaré.

Leśniewski took a firm stand in the dispute concerning universals − he rejected the

[9] Sprzyjało to zanikowi poczucia różnicy między naukami matematycznymi pojmowanymi jako teorie dedukcyjne, służące do ujęcia w prawa możliwie ścisłe różnorodnej rzeczywistości świata, a takimi niesprzecznymi systemami dedukcyjnymi, które zabezpieczają wprawdzie możność otrzymania na ich gruncie obfitości wciąż nowych twierdzeń, odznaczających się jednak jednocześnie brakiem jakichkolwiek łączących je z rzeczywistością walorów intuicyjno-naukowych. ([17], p. 166)

[10] Ich sähe keinen Widerspruch darin, wenn ich behaupten wollte, dass ich eben deshalb beim Aufbau meines Systems einen ziemlich radikalen «Formalismus» treibe, weil ich ein versteckter «Intuitionist» bin: indem ich mich beim Darstellen von verschiedenen deduktiven Theorien bemühe, in einer Reihe sinnvolle Sätze eine Reihe von Gedanken auszudrücken, welche ich über dieses oder jedes Thema hege, und die einen Sätze aus den anderen Sätzen auf eine Weise abzuleiten, die mit den Schlussweisen harmonisieren würden, welche ich «intuitiv» als für mich bindend betrachte [...]. ([18], p. 78).

existence of any ideal and general objects. In the paper “Krytyka logicznej zasady wyłączonego środka” [Critique of the logical principle of excluded middle] (cf. [16]) he gave the proof of non-existence of such objects that became very popular in Poland. In the proof the concept of a feature as well as the principle of excluded middle and the principle of contradiction were used. It was quoted − with some modifications − by Kotarbiński in the paper “Sprawa istnienia przedmiotów idealnych” [The problem of existence of ideal objects] (cf. [11]) and repeated in his book *Elementy teorji poznania, logiki formalnej i metodologji nauk* [Elements of the theory of cognition, formal logic and methodology] (cf. [12]). It became one of the justifications of reism propagated by him. Leśniewski return to his proof in the work “O podstawach matematyki” [On the foundations of mathematics] (cf. [17], pp. 183−184) where he gave a new version of it in which the concept of “feature” does not appear. The proof was preceded by the following explanations:

> At the time I wrote that passage [Leśniewski says about the appropriate fragment of his [16] − our remark] I believed that there are in existence in this world so called features and so called relations, as two special kinds of objects, and I felt no scruples about using the expressions ‘feature’ and ‘relations’. It is a long time since I believed in the cxistence of objects which are features , or in the existence of objects which are relations and now nothing induces me to believe in the existence of such objects […] and in situations of a more ‘delicate’ character I do not use the expressions ‘feature’ and ‘relation’ without the application of various extensive precautions and circumlocutions. I also have no inclination at present − considering the possibility of various interpretational misunderstandings − to ascribe this or that opinion on the question of ‘general objects’ to the authors mentioned in the passage mentioned above.[11] ([21], p. 198).

3 Alfred Tarski − Nominalism (?)

A follower of nominalism was also Alfred Tarski (1901−1983). This pronominalistic attitude was the source of the fact that in the interwar period he treated language as a set of

[11] W czasie, gdy ustęp ten [chodzi tu o stosowny fragment pracy [16] − uwaga moja, R.M.] pisałem, wierzyłem, iż istnieją na świecie tzw. cechy i tzw. stosunki jako dwa specjalne rodzaje przedmiotów, i nie odczuwałem żadnych skrupułów przy posługiwaniu się wyrazami „cecha” i „stosunek”. Obecnie nie wierzę już od dawna w istnienie przedmiotów będących cechami, ani też w istnienie przedmiotów będących stosunkami, nic mnie też nie skłania do wierzenia w istnienie takich przedmiotów [...], wyrazami zaś „cecha” i „stosunek” staram się w sytuacjach o cokolwiek „delikatniejszym” charakterze nie posługiwać bez daleko idących ostrożności i omówień. Nie mam dziś także skłonności - wobec możliwości rozmaitych nieporozumień interpretacyjnych - do przypisywania tych lub innych poglądów w sprawie „przedmiotów ogólnych” tym lub innym z autorów, wymienionym w ustępie wyżej przytoczonym. ([17], p. 183)

sentences understood in a strictly nominalistic way as physical objects. However his sympathies towards nominalism were in fact stronger. Mostowski wrote about this in the following way:

> Tarski, in oral discussions, has often indicated his sympathies with nominalism. While he never accepted the »reism« of Tadeusz Kotarbiński, he was certainly attracted to it in the early phase of his work. However, the set-theoretical methods that form the basis of his logical and mathematical studies compel him constantly to use the abstract and general notions that a nominalist seeks to avoid. In the absence of more extensive publications by Tarski on philosophical subjects, this conflict appears to have remained unresolved ([34], p. 81).

Tarski's pronominalistic attitude is confirmed in various sources. Firstly, it was Tarski's remark (preserved on a tape cassette) made during the symposium organised by the Association for Symbolic Logic and the American Philosophical Association, held in Chicago on 29th−30th April 1965, and dedicated to philosophical implications of Gödel's incompleteness theorem. Tarski said:

> I happen to be, you know, a much more extreme anti-Platonist. [...] However, I represent this very [c]rude, naive kind of anti-Platonism, one thing which I would describe as materialism, or nominalism with some materialistic taint, and it is very difficult for a man to live his whole life with this philosophical attitude, especially if he is a mathematician, especially if for some reasons he has a hobby which is called set theory ([9], p. 52).

Fefermans' book [9] contains more similar words concerning Tarski himself or other people's opinions about Tarski. These opinions were expressed on Tarski's 70th birthday celebrations and remembered by Chihara, Chateaubriand and the Fefermans:

> I am a nominalist. This is a very deep conviction of mine. It is so deep, indeed, that even after my third reincarnation, I will still be a nominalist. [...] People have asked me, »How can you, a nominalist, do work in set theory and logic, which are theories about things you do not believe in?« ... I believe that there is a value even in fairy tales.
>
> [I am] a tortured nominalist.
>
> Elsewhere Tarski has said more specifically that he subscribed to reism or concretism (a kind of physicalistic nominalism) of his teacher Tadeusz Kotarbiński ([9], p. 52).

Also Tarski's letter to Woodger, dated 21st November 1948, testifies to the importance he

attached to nominalism:

> The problem of constructing nominalistic logic and mathematics has intensively interested me for many-many years. Mathematics – at least the so-called classical mathematics – is at present an indispensable tool for scientific research in empirical sciences. The main problem for me is whether this tool can be interpreted nominalistically or replaced by another nominalistic tool which should be adequate for the same purposes ([28], p. 147).

On many occasions Tarski stressed his sympathies towards Kotarbiński's reism and physicalism. He also translated into English (together with David Rynin) Kotarbiński's work "Zasadnicze myśli pansomatyzmu" [The Fundamental Ideas of Pansomatism] (cf. [13]). The translation was published in *Mind*, one of the most important English periodicals dedicated to philosophy. It was included in Tarski's *Collected Works* [40].[12]

More details about Tarski's sympathies and inclinations towards nominalism can be found in the recently discovered protocols of Carnap from the discussions conducted at Harvard in the academic year 1940/41. Besides Carnap the other participants were Tarski and Quine as well as – occasionally – Russell.

In the protocol of 10th January 1941 Carnap wrote down the following remarks concerning nominalism and finitism:

> Tarski: I understand basically only languages which satisfy the following conditions:
> 1. Finite number of individuals;
> 2. Realistic (Kotarbinski): the individuals are physical things;
> 3. Non-platonic: there are only variables for individuals (things) not for universals (classes and so on) [13] ([27], p. 342).

Mancosu notices ([27], p. 343) a mistake: instead of 'realistic' it should be 'reistic,' which is confirmed by the reference to Kotarbiński.

Carnap's notes also contain the following exchange of views:

> I [Carnap]: Should we construct the language of science with or without types?

[12] It also testifies to Kotarbiński's strong influence on Tarski.

[13] Tarski: Ich verstehe im Grunde nur eine Sprache die folgende Bedingungen erfüllt: [1] Finite Anzahl der Individuen; [2] Realistisch (Kotarbiński): Die Individuen sind physikalische Dinge; [3] Nicht-platonisch: Es kommen nur Variable für Individuen (Dinge) vor, nicht für Universalien (Klassen usw.).

He [Tarski]: Perhaps something else will emerge. One would hope and perhaps conjecture that the whole general set theory, however beautiful it is, will in the future disappear. With the higher types Platonism begins. The tendencies of Chwistek and others (`Nominalism') of speaking only of what can be named are healthy. The problem is only how to find a good implementation.[14] ([27], p. 334)

Of special interest – in the context of the problem of passing from the systems of the theory of classes – is also Carnap's summary of his conversation with Tarski on 12th February 1941:

The Warsaw logicians, especially Leśniewski and Kotarbiński saw a system like PM (but with simple type theory) as the obvious system form. This restriction influenced strongly all the disciples; including Tarski until the 'Concept of Truth' (where the finiteness of the levels is implicitly assumed and neither transfinite types nor systems without types are taken into consideration; they are discussed only in the Postscript added later). Then Tarski realized that in set theory one uses with great success a different system form. So he eventually came to see this type-free system form as more natural and simpler ([27], p. 335).[15]

One should notice that Tarski's research practice, in particular his investigations concerning set theory or the theory of models, contradicted in fact his nominalism to a certain extent and would rather suggest that he was a follower of Platonism (this explains the question mark in the title of this section). This discrepancy can be explained by the spirit and ideological canon of the Polish School. According to them, research should not be limited by any *a priori* philosophical foundations and all correct methods should be allowed and applied.

[14] Ich: Sollen wir vielleicht die Sprache der Wissenschaften mit oder ohne Typen machen? Er: Vielleicht wird sich etwas ganz Anderes entwickeln. Es wäre zu wünschen und vielleicht zu vermuten, dass die ganze allgemeine Mengenlehre, so schön sie auch ist, in der Zukunft verschwinden wird. Mit den höheren Stufen fängt der Platonismus an. Die Tendenzen von Chwistek und anderen (»Nominalismus«), nur über Bezeichenbaren zu sprechen, sind gesund. Problem nur, wie gute Durchführung zu finden.

[15] *Die Warschauer Logiker*, besonders Leśniewski und Kotarbiński, sahen ein System wie PM (aber mit einfacher Typentheorie) ganz selbstverständlich als die Systemform an. Diese Beschränkung wirkte stark suggestiv auf alle Schüler; auf T. selbst noch bis zu »Wahrheitsbegriff« (wo weder transfinite Stufen noch stufenloses System betrachtet wird, und Endlichkeit der Stufen stillschweigend vorausgesetzt wird, erst im später hinzugefügten Anhang werden sie besprochen). Dann aber sah T., dass in der Mengenlehre mit grossem Erfolg eine ganz andere Systemform verwendet wird. So kam er schliesslich dazu, diese stufenlose Systemform als natürlicher und einfacher zu sehen.

4 Tadeusz Kotarbiński − Reism

Presenting philosophical views of Tarski we have mentioned Tadeusz Kotarbiński (1886−1981) and his doctrine of reism to which Tarski referred. Let us say now something more about this.

Reism is by Kotarbiński both a semantical and an ontological doctrine, moreover both levels are in a certain sense parallel. Kotarbiński admitted that developing reism he used some logical ideas of Leśniewski explained by the latter in his system of the calculus of names called ontology. In the Preface to *Elementy teorii poznania, logiki formalnej i metodologii nauk* [Elements of epistemology, formal logic and methodology of science] (cf. [14]) Kotarbiński wrote:

> Still, I have learnt most things from Prof. Dr Stanisław Leśniewski. I admit that in many places of the book. And they are the most important and clearest points. Besides I admit that all my thoughts are deeply saturated with the influences of that extraordinary mind whose precious gifts I have used, thanks to good luck, almost every day for a number of years. I am undoubtedly a disciple of my colleague Leśniewski whom here I thank cordially and respectfully for all that he has ever taught me.[16] ([15], pp. 9−10).

Add that Leśniewski himself valued his collaboration with Kotarbiński. He admitted that he owed him a lot (see for example [38], p. 93).

The source of Kotarbiński's reism were his doubts concerning the existence of properties and other ideal objects. He expressed them for the first time in his paper "Sprawa istnienia przedmiotów idealnych" [The problem of existence of ideal objects] (cf. [11]). He criticized there conceptions assuming the existence of ideal objects. He wrote that there were no foundations to assume the existence of such objects. He tried to show that there were no imaginary (only conceivable) objects, no mathematical objects; there were no types (universals), features, relations, intentional objects, thinking processes and psychological contents.

[16] Najwięcej wszelako nauczyłem się od prof. dra Stanisława Leśniewskiego. W wielu miejscach książki wyraźnie z tego zdaję sprawę. Ale to są punkty najważniejsze i najwyraźniejsze. Poza tym, przyznaję, cała myśl moja przesycona jest do głębi wpływami tego niezwykłego umysłu, z którego bezcennych darów los przychylny pozwolił mi przez szereg lat korzystać w obcowaniu niemal codziennym. Jestem niewątpliwie uczniem kolegi Leśniewskiego, któremu na tym miejscu serdecznie i z głębokim szacunkiem dziękuję za wszystko, czego mnie kiedykolwiek nauczył. ([14], pp. 9−10)

Reism was explained by Kotarbinski in his book *Elementy* [Elements] (cf. [12] and [14]) and in various papers. Reism in the ontological sense can be reduced to the following two theses: (1) every object is a thing, (2) no object is a state, a relation, a feature[17]. Kotarbinski assumes also that things are bodies, and thus extensive beings existing in time and space. Therefore, we are dealing with somatism strengthened to become pansomatism – there are only bodies. This distinguishes reism from other concretisms, for example from the concretism of Leibniz who towards the end of his life assumed that there were only concrete entities (note that this concretism was of spiritualistic nature because those concrete entities were spiritual monads). Reism can be seen as a certain interpretation of Leśniewski's ontology (the latter was not a reist although he was a nominalist).

Reism faces various difficulties when applied to logic and mathematics. Using the language of reism one can speak about sets in a distributive sense that is fundamental for set theory, on which in turn the whole building of mathematics is constructed, but only providing that those statements refer to the elements of these sets. Hence it allows us to develop the elementary algebra of sets but not to define, for instance the concept of finite or infinite set. However, it is not sufficient for mathematics. Leśniewski was aware of these difficulties and proposed to use the concept of a set in a collective sense (mereological) − such an approach does not allow realising all that mathematicians expect of set theory. It should be added that reism had numerous followers, the greatest one being Alfred Tarski.[18] Furthermore, reism, thanks to its logical tools, allows achieving more than any other nominalism.

In the ontology of mathematics Kotarbiński proclaimed himself in favour of nominalism. In *Elementy* he wrote:

> In this variety of opinions, let us single out, and declare for, the position of nominalism. […] no object is a number, and […] neither arithmetic, nor the theory of numbers, nor − *a fortiori* − mathematics in general build statements which might strictly be called statements about numbers in the same sense in which zoology makes statements about animals.[19] ([15], p. 317)

[17] A clear reference to the four categories proposed by W. Wundt can be seen here.

[18] It is worth quoting the words of Andrzej Mostowski uttered after returning from a conference dedicated to the foundations of set theory: "Just imagine that there I sighed for reism. The presented conceptions resulted from so breakneck speculations, so unattainable for intuition and so incomprehensible that reism seemed to be an oasis where one can breathe fresh air." (cf. [11], p. 73)

[19] W tym nadmiarze rozmaitych stanowisk niechaj nam wolno będzie wyróżnić stanowisko nominalizmu i przy nim się opowiedzieć. [...] żaden przedmiot nie jest liczbą i [...] ani arytmetyka, ani tzw. „teoria liczb", ani tym bardziej matematyka w ogóle nie budują zdań, które by można nazwać ściśle zdaniami o liczbach w tym sensie, w jakim np. zoologia mówi o zwierzętach ([14], p. 373).

Mathematics speaks about all things – and hence, its universality.

Kotarbiński − firmly refuting the conception that mathematics investigates a certain world of ideal objects dependent on time, space and cognitive mind − did not follow any concrete conception. He stated that mathematics can be characterised in at least three ways:

(1) as the body of systems in which theorems are justified only in a deductive way and 'the theorems are formulated correctly as statements containing only the following types of signs − variables, connectives, what are called 'names of numbers", 'names of sets", 'names of figures", or terms defined by such signs, names of relations (such as 'greater than", 'equal to", etc,) and finally punctuation marks and signs informing about the role of the remaining signs' ([15], p. 322)[20] – mathematics thus understood embraces the whole formal logic (in its propositions these 'names' do not occur) and the so-called proper mathematics;

(2) as proper mathematics or mathematics in a narrower sense, which is characterised by the fact that those 'names' occur in its thesis;

(3) as a science that is characterised like proper mathematics but adding the condition that its propositions have the feature of apriority, i.e., its axioms are assigned the feature of obviousness, and justifying its theorems we do not refer to empirical data.

Add that according to Kotarbiński nominalism is consistent with the thesis about aproristic character of mathematics a science.

5 Leon Chwistek − Nominalism

Talking about nominalism one should mention also Leon Chwistek (1884−1944). Though he did not belong directly to Lvov-Warsaw School, he went always along his own paths being a "separate" scholar, nevertheless his conceptions were important.

Chwistek declared himself as a nominalist. According to him the subject of deductive sciences, hence also of mathematics, are expressions constructed in them according to accepted rules of construction. Hence the subject of mathematics are not ideal objects like points, lines, numbers or sets. Expressions being subjects of mathematics are physical objects given us in experience. They can be transformed according to accepted rules. In every

[20] których twierdzenia wypowiada się poprawnie w zdaniach, zawierających tylko następujące rodzaje znaków: symbole zmienne, spójniki, tzw. »nazwy liczb«, tzw. »nazwy zbio- rów«, tzw. »nazwy figur«, lub terminy przez takie znaki zdefiniowane, dalej terminy stosunkowe, jak »większy«, »równy« itp., wreszcie znaki przestankowe oraz znaki informujące o roli pozostałych znaków. ([14], p. 379).

system one accepts such rules as well as some expressions that play the role of axioms and that form the base on which theorems are deduced. Transformation rules and axioms are chosen in such a way that expressions could be interpreted as descriptions of considered states of affairs. To be able to apply deductive theories to specific sciences and generally to perceive concrete areas of reality, the elements of the latter should be schematised.

In particular geometry is − according to Chwistek an experimental discipline. In Chapter VIII of *Granice nauki* [Limits of Science] he wrote:

> Geometry is an experimental science. It depends upon the measurement of segments, angles, and areas. The Egyptians conceived it in this way and it has remained essentially the same up to this very day. Today what is generally regarded as geometry, i.e. what is included in textbooks, is the peculiar mixture of experimental geometry and the geometrical metaphysics which was inherited from the Greeks as Euclid's *Elements*.[21] ([5], p. 170)

The rise of the systems of non-Euclidean geometry of Bolyai, Gauss and Lobachevsky in the 19th century − regarded by Chwistek as the most important achievement in exact sciences − abolished in his opinion Kant's idealism. These geometries showed that, for example, the concept of a straight line is not of an objective character, but depends on the accepted axioms. It may suggest that conventionalism is the proper philosophy for geometry. Indeed, in his first works, e.g. the paper "Trzy odczyty odnoszące się do pojęcia istnienia" [Three Talks Concerning the Concept of Existence] (see [3]), he states that the existence of systems of non-Euclidean geometry, which are consistent, refutes the thesis of the *a priori* character of geometry. It seems that he would tend to accept conventionalism, although he does not state this explicitly. However in *Granice nauki* [4] he explicitly and categorically rejected conventionalism claiming that geometry − similarly as all other fundamental experimental sciences − should be based on the theory of expressions. This is because conventionalism introduces hypothetical entities, as was the case in John Stuart Mill's works or later Poincaré's, a promoter of this direction. Chwistek wrote:

[21] Geometria jest nauką doświadczalną. Polega ona na mierzeniu odcinków, kątów i powierzchni. Tak pojmowali ją Egipcjanie i taką pozostała w istocie swojej do dzisiaj. To, co uważa się powszechnie za geometrię za naszych czasów, tj. to, o czym pisze się w podręcznikach, jest osobliwą mieszaniną geometrii doświadczalnej i metafizyki geometrycznej, którą pozostawili nam w spadku Grecy pod postacią elementów Euklidesa. ([4], p. 190; see also [6], p. 170)

It seems that it is impossible to attain a general concept of geometry without using formulae. It is therefore clear that the conception of geometry as the science of ideal spatial constructions must be nullified… . To speak of different four-dimensional space-times it is necessary to employ five-dimensional spacetime. It is clear that all this has only as much meaning as do mathematical formulae.[22] ([5], pp. 186−187).

In a similar way as geometry one should treat arithmetic, mathematical analysis and other mathematical theories obtaining in this way consequently their nominalistic interpretations.

6 Kazimierz Ajdukiewicz and Ontology of Mathematics

Let us turn back to Lvov-Warsaw School and consider ontological views of Kazimierz Ajdukiewicz (1890−1963). He considered the ontology of mathematics and logic in his *Habilitationsschrift* entitled *Z metodologii nauk dedukcyjnych* [From the Methodology of Deductive Sciences] from 1921 (cf. [1]) in which he discussed the problem of existence, in particular the problem what does it mean "to exist" in deductive sciences. He wrote there:

An analysis of meaning of the word 'exist' as used in deductive theories does not amount to the problem: what kind of existence is among the attributes of existing objects of deductive theories; our own position permits us to doubt whether any kind of being at all is among the attributes of these objects. Our problem then is not the question what kind of being is attributable to objects under discussion, but the question what is the meaning of the word 'exist' as used in deductive theories. It may be that it is being used quite erroneously and has nothing at all to do with existence.[23] ([2], p. 34)

Ajdukiewicz argued in the considered work that the existence in deductive sciences cannot be identified with consistency and that consistency is neither sufficient nor necessary condition of existence. He claimed that the necessary conditions of existence are: (I) being included into the domain of the given theory, and (II) consistency:

[22] Okazuje się, że dotarcie do ogólnego pojęcia geometrii bez formuł jest niemożliwe. Jasne jest, że idąc tą drogą, musimy dojść do unicestwienia geometrii jako nauki o idealnych utworach przestrzennych. [...] Żeby mówić o różnych czterowymiarowych czasoprzestrzeniach, musimy się odwołać do czasoprzestrzeni pięciowymiarowej. Jest jasne, że wszystko to ma tyle sensu, ile zawierają go formuły matematyczne. ([4], pp. 186−187)

[23] Analiza znaczenia wyrazu „istnieć" w naukach dedukcyjnych nie jest zatem równoznaczna z zagadnieniem: jaki rodzaj istnienia przysługuje istniejącym przedmiotom nauk dedukcyjnych; problemat nasz pozwala nam w ogóle wątpić o tym, czy jakikolwiek rodzaj bytu przedmiotom tym przysługuje. Kwestią naszą zatem nie jest pytanie, co za rodzaj bytu mają przedmioty przez nas rozważane, ale co znaczy wyraz „istnieć" w naukach dedukcyjnych. Być może, że jest on całkiem mylnie używany i nie ma z istnieniem nic wspólnego. ([1], p. 46)

My contention is, namely, that for an object *p* defined by Ω (*p*) to exist it is necessary that *p* be an element of the domain of the given theory, in other words that Ω (*p*) entailed *A*(*p*) [...].

In order to exist an object must, therefore, satisfy another requirement – besides the above condition of being an element of the domain of the theory – scil. its definition must not have any consequences inconsistent with the consequences of *A(p)*. [...]

Objects which do not satisfy either the first or the second requirement, do not exist, are nonexisting. From existing and nonexisting objects we ought to distinguish objects which are possible in the given theory.[24] ([2], pp. 42−43)

Ajdukiewicz comes to the conclusion that if an object is to exist it must satisfy the requirements (I) and (II) as well as 'not restrict the domain of possible objects'[25] ([2], p. 44) in the given theory. And he concludes his considerations in the following way:

> In the deductive sciences we do not speak of existence in absolute sense but only relatively to a given system. For there exist Euclidean straight lines and non-Euclidean straight lines; however, both cannot co-exist and their co-existence would be a consequence of their existence if this word were taken in either case in the absolute sense. We may only speak of existence in a system as we speak of inclusion in a domain. Nevertheless it is possible to construct a 'universe' consisting of the domains of several compatible theories, thus forming a system whose axioms would be all axioms of all compatible theories. We could then speak of absolute existence, not quite absolute, though, since it would be possible by choosing various theories, to construct many such 'universes,' self-compatible but mutually exclusive.[26] ([2], p. 45)

[24] Twierdzę mianowicie, że koniecznym warunkiem na to, by przedmiot określony przez Ω (*p*) istniał, jest iżby przedmiot *p* należał do zakresu danej teorii, czyli iżby z Ω (p) wynikało *A*(*p*) [...].

Musi tedy przedmiot na to, aby istniał, spełniać prócz pierwszego (wyżej wymienionego warunku zawierania się) warunek drugi, musi mianowicie jego określenie nie posiadać następstw sprzecznych z następstwami *A*(*p*). [...]

Przedmioty, które nie czynią zadość pierwszemu albo drugiemu warunkowi, nie istnieją i są nieistniejące. Prócz przedmiotów istniejących i nieistniejących należy jeszcze rozróżnić, naszym zdaniem, przedmioty możliwe w danej teorii. ([1], pp. 59−60)

[25] nie ograniczał [on] zakresu przedmiotów możliwych ([1], p. 62)

[26] O istnieniu bezwzględnym w naukach dedukcyjnych nie mówimy wcale. Zawsze tylko o istnieniu w pewnym systemie. Wszakże istnieją i proste euklidesowe, i nieeuklidesowe, obie nie mogą jednak współistnieć, a współistnienie ich byłoby konsekwencją ich istnienia, gdyby ten wyraz wziąć w odniesieniu do obu w tym samym sensie bezwzględnym. Można więc mówić tylko o istnieniu w pewnym systemie, podobnie jak o zawieraniu się tylko w pewnym zakresie. Niemniej jednak można utworzyć „uniwersum" z zakresów kilku zgodnych z sobą teorii, tworząc system, którego aksjomaty byłyby wszystkimi aksjomatami wszystkich teorii zgodnych. Można by wtedy mówić o istnieniu bezwzględnym, jakkolwiek niezupełnie bezwzględnym, bo można

7 Andrzej Mostowski − Nominalism, Reism, Constructivism

Let us finish our considerations by presenting ontological views of Andrzej Mostowski (1913−1975). He is usually treated as a representative of the second generation of Lvov-Warsaw School. He was disciple of Tarski, some influence of Alfred Lindenbaum on him can be seen.

Mostowski inherited from Tarski general philosophical views, in particular tendency towards empiricism and apparent respect for nominalism. His sympathies were also, as it seems, with Kotarbiński's reism, i.e., the view that there exist only individual physical things. However he avoided in his logical and mathematical works explicit philosophical declarations. Nevertheless there are some exceptions from that tendency.

In the paper "A Classification of Logical Systems" ([30]) Mostowski assumes a certain philosophical presumption on the analysed logical systems. Declaring at the beginning of his work that "The subject itself as well as the method of its presentation will be of a mathematical rather than philosophical character" ([30], p. 245) he openly states:

> Although our investigations will be purely formal we shall nevertheless accept a definitive philosophical point of view with respect to logical systems. We shall not consider logical systems as void schemata deprived of any interpretation. On the contrary we shall assume the objective existence of a kind of »mathematical reality« (e.g. of the set of all integers or the set of all real numbers). By objective existence we mean existence independently of all linguistic constructions ([31], pp. 246−247).

The task of logical systems is − according to Mostowski − just to describe that "mathematical reality". Consequently, every sentence of logic is equipped with a certain meaning – it says that mathematical reality is entitled to have this or that property. The fact that there exist true sentences that are unprovable in a given system − what is a consequence of Gödel's incompleteness theorems − can be explained by the bigger complexity of the properties of this "mathematical reality" than the complexity of the properties that can be deduced from axioms by the accepted inference rules. Mostowski concludes in a characteristic way:

We do not intend to defend the philosophical correctness or even the philosophical

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by, dobierając rozmaite teorie, potworzyć wiele takich „uniwersów" w sobie zgodnych, lecz między sobą wykluczających się. ([1], p. 63)

> acceptability of the point of view here described. It is evident that it is entirely opposite to the point of view of nominalism and related trends ([30], pp. 247).

One can easily see here certain tension between aforementioned inclinations towards nominalism and his concrete logical and mathematical investigations.

Philosophical questions, in particular ontological ones, appear in Mostowski's papers devoted to set theory. Considering Gödel's and Cohen's results on consistency and independence of Axiom of Choice and Continuum Hypothesis Mostowski claimed that they can be treated as "one of the most important arguments against mathematical Platonism" ([34], p. 176). After Gödel's and Cohen's results it is possible to construct consistent but mutually inconsistent set theories. If such theories are constructed "we shall be forced to admit that in the match between Platonism and formalism the latter has again scored one point" ([34], p. 182). Since metamathematical results on set theory do not provide decisions concerning the way of existence of sets, and in general of objects of mathematics, and consequently the controversy between formalism and Platonism, Mostowski concludes in the paper "Sets" ([35]):

> Whatever the final outcome of the fight between these two opposing trends will be, it is obvious that we should concentrate on the study of concepts which seem perfectly clear and perspicuous to us. ([35], p. 28)

At another place however he wrote that "the ultimate formulation of axioms of set theory should be preceded by a discussion of the fundamental assumptions of this theory, including the constructive standpoint" ([31], p. 20).

Mostowski had a keen interest in constructivism. However it should be added that he was more interested in its aims than in proposed solutions (see for example [32], p. 192). In the monograph *Logika matematyczna* [Mathematical Logic] ([29]) he wrote even:

> I am inclined to think that a satisfactory solution of the foundations of mathematics will happen on the way shown by constructivism or a similar direction. However, one cannot now write a textbook of logic on this basis.[27] ([29], p. VI)

[27] Jestem skłonny mniemać, że zadowalające rozstrzygnięcie zagadnienia podstaw matematyki nastąpi na drodze wskazanej przez konstruktywizm lub kierunek do niego zbliżony. Na tej jednak podstawie nie można by już teraz napisać podręcznika logiki.

Mostowski saw advantages of constructivism in the fact that:

> [...] it wants to inquire into the nature of mathematical entities and to find a justification for the general laws which govern them, whereas Platonism takes these laws as granted without any further discussion ([32], p. 192).

According to Mostowski constructivistic conceptions are closer to nominalism than to Platonism. This implies that constructivism does not accept general concepts of mathematics as being given but attempts to construct them. "This leads to the result that one can identify mathematical concepts with their definitions" ([32], p. 178). In arithmetic constructivism allows us to give up assuming actual infinity or to use solutions requiring only the nominalist approach. Whereas one of the advantages of nominalism is that many important mathematical theories have been satisfactorily reconstructed on the nominalist basis, and these reconstructions have turned out to be equivalent to the classical theories.

Mostowski was aware of some limitations of constructive methods in mathematics and of the fact that they do not suffice (cf. for example [35], pp. 29−32). Nevertheless he investigated principles of constructivism. He was of the opinion that sometimes constructivism is philosophically more satisfactory − this is the case of arithmetic or of applied mathematics where it seems to reveal new promising perspectives.

It should be noticed that Mostowski considered constructivism in a way connected with the classical point of view. Hence he was not connected with pure constructivism of Brouwer, Heyting and other intuitionists. In fact he represented rather certain combination of constructivism and set-theoretical program. This combination formed according to him the base on which the foundations of mathematics should be developed.

8 Conclusion

The described panorama of ontological views concerning logic and mathematics of representatives of Lvov-Warsaw School shows that one does not find here any dominating position, just opposite, there is a full range of positions from Platonism through constructivism till nominalism. What was the reason of that? First of all it should be stressed that philosophical views were formulated in Lvov-Warsaw School on the margin of proper logical or metamathematical investigations. One was convinced that mathematical and logical

investigations should not bounded by any *a priori* philosophical assumptions. Mathematics and logic should be autonomous and neutral with respect to philosophy. Hence opinions about philosophical aspects of logic or mathematics were rather fragmentary and incomplete, they concerned first of all particular issues connected with problems actually studied. Formulated remarks were often simply comments to concrete technical results from the foundations of mathematics or logic. Exceptions were here Leśniewski and Chwistek whose logical investigations were a consequence and result of some philosophical considerations.

Philosophical views were generally treated as private matters. They should not bound the research activity in logic, mathematics or the foundations of mathematics and during the very investigations of concrete mathematical or logical problems should be suspended. Moreover it happed that declared philosophical views were in fact not compatible with research practice. It can be easily seen for example by Tarski who declared himself as nominalist but simultaneously in his research practice used without any restrictions infinititistic methods far from what has been accepted by nominalists.

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