Many-Valued Logics in the Iberian Peninsula

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Abstract The problems of uncertainty, imprecision and vagueness have been discussed for many years. These problems have been major topics in philosophical circles with much debate, in particular, about the nature of vagueness and the ability of traditional Boolean logic to cope with concepts and perceptions that are imprecise or vague [39, 46]. The Fuzzy Logic (which is usually translated into Castilian by "Lógica Borrosa", or "Lógica Difusa", but also by "Lógica Heurística") can be considered a bypass-valued logics (Multi-valued Logic, MVL, its acronym in English). It is founded on, and is closely related to-Fuzzy Sets Theory, and successfully applied on Fuzzy Systems. You might think that fuzzy logic is quite recent and what has worked for a short time, but its origins date back at least to the Greek philosophers and especially Plato (428-347 BC). It even seems plausible to trace their origins in China and India [24]. Because it seems that they were the pioneers in considering that there may be varying degrees of truth and falsehood. In case of colors, for example, between white and black there is a whole infinite scale: the shades of gray. Some recent theorems show that in principle fuzzy logic can be used to model any continuous system, be it based in AI, or physics, or biology, or economics, etc. [40, 41]. Investigators in many fields may find that fuzzy, commonsense models are more useful, and they are many times standard mathematical. We analyze here the history and development of this problem: Fuzziness, or "Borrosidad" (in Castilian), essential to work with Uncertainty.

Our purpose is contributing to the search for the origins of many-valued logics (MVL, by acronym), its relation with G. W. Leibniz, and, within MVLs, take the name "Fuzzy Logic" [25, 28].

It is also our goal how was introduced Jan Lukasiewicz, Lofti A. Zadeh, and many-valued logics in our Iberian Peninsula, in the world philosophical universe.

Keywords Aristotle. G. W. Leibniz. Non-Classical Logics. Many-Valued Logics. Fuzzy Logic. Fuzzy Measures and Integrals. Applied Logic in the Iberian Peninsula.

Mathematics Subject Classification (2000) 01A20. 01A55. 01A60. 03A05. 03B50. 03B52. 03B60. 03B80. 03E72. 03E75. 03G20. 08E72.

To Prof. Dr. Solomon Marcus (1925-2016),
who recently passed away.

1 INTRODUCTION

As we know, logic is the study of the structure and principles of correct reasoning, and more specifically, attempts to establish the principles that guarantee the validity of deductive arguments [20, 26]. The central concept of validity is for logic, because when we affirm the validity of an argument are saying that it is impossible that its conclusion is false if its premises are true.

Propositions are descriptions of the world, that is, are affirmations or denials of events in various possible worlds, of which the "real world" is just one of them [34, 35, 49, 50]. There is a long philosophical tradition of distinguishing between truth necessary (a priori or "logical") and facts "contingent" (a posteriori or "factual").

Both have really led the two concepts of logical truth, without being opposed to each other, are quite different: the conception of truth as coherence, and the conception of truth as correspondence. According to the point of view of consistency, a proposition is true or false depending on their relationship with respect to a given set of propositions, because of the rules of that system. Under the terms of correspondence, a proposition is true or false, if it agrees with reality, that is, the fact referred to [36, 38].

To further enhance the complexity of the problem, not only analyze trueness or falsity of propositions, but also of theories, ideas and models [13, 25, 31]. And so, we allow new and different conception of truth.

The basic idea underlying all these approaches is that of an intrinsic dichotomy between true and false. This opposition implies the validity of two fundamental laws of classical logic:

- Principle of excluded middle: Every proposition is true or false, and there is another possibility.
- Principle of non-contradiction: No statement is true and false simultaneously.

Such fundamental ideas produce some series of paradoxes and dissatisfaction that is based on the need to overcome this strict truth-bivalence of classical logic.

Searching for the origins could lead too far and eventually disperse, which, as we know is not very convenient for a job pretending to be research. So we will refer to these first signs that appear in the East (China, India…), and then we may analyze the problem of "future contingents", treated by Aristotle in *Peri hermeneias.*

"For a man may predict an event ten thousand years beforehand, and another may predict the reverse; that which was truly predicted at the moment in the past will of necessity take place in the fullness of time" (Aristotle, *Peri Hermeneias,* ch. 9).

About *Future Contingent Propositions*, we must remember that they are statements about states of affairs in the future that are neither necessarily true nor necessarily false. Suppose that a sea-battle will not be fought tomorrow. Then it was also true yesterday (and the week before, and last year) that it will not be fought, since any true statement about the case that will be was also true in the past. But all past truths are now necessary truths; therefore it is now necessarily true that the battle will not be fought, and thus the

statement that it will be fought is necessarily false. Therefore it is not possible that the battle will be fought. In general, if something will not be the case, it is not possible for it to be the case [37, 47, 60].

As we know, although the starting point of Leibniz "calculus universalis" were Stagirite´s theories, Leibniz ends to be dependent from the ideas of Aristotle, to finally develop its own axiomatic system, a more general type, based on applying the Combinatorial Instrument to syllogistic [30, 56].

That issue (Future Contingent´s problem, with variations) would be then crucial in medieval times, as during the Scholasticism, with William of Ockham, and Duns Scotus, or Richard of Lavenham, among others, looked at from different point of views, for its relationships with Determinism and `Divine Foreknowledge´. Then, this issue is taken up by Spanish Jesuit F. Luis de Molina (and the famous controversy `De Auxiliis´ maintained with the Dominican Fray Domingo Báñez), or Francisco Suarez, and even the great polymath G. W. Leibniz dedicated his time [25, 26, 34].

The controversial "De Auxiliis" involves two key works: the *Concordia*, from the Jesuit Father Luis de Molina (1535-1600), and the *Apology*, from Fray Domingo Báñez (1528-1604), a Dominican School of Salamanca and San Esteban´s Convent. In essence, it represented the possible antagonism between free will of humans and efficacy of Divine Grace. In short: Is Omniscience and Omnipotence compatible with the man´s freedom? The discussion took a particularly interesting way during the Middle Age. In this period philosophy was interconnected with theology. And one of the most important theological issues was precisely the problem of future contingents, in its direct relationship with Christian doctrine. According to this tradition, related with the Divine Foreknowledge. It includes knowledge of future possibilities to be made by human beings. But this assumption seems to lead to a simple argument. It leads from foreknowledge to the need of future events: now known as God and I will take the decision tomorrow, it´s true that my choice of morning is given. My choice then, it seems necessary but not free. Therefore, there appears to be no basis for claim that we have freedom of choice among alternatives. The conclusion, however, would violate the idea of human freedom and of moral responsibility [27, 57, 58].

Even then there is a dark time for the logic, and reappearing in the nineteenth century, philosophers and mathematicians such as George Cantor, Augustus De Morgan and George Boole, Gottlob Frege... There was born the new set theory, now called "classic", but then also had terrible enemies, as the then almighty Leopold Kronecker, who from his professorship in Berlin did everything possible to hinder the work of Cantor, and the rise of those new ideas.

As Kluge said, Frege's logico-mathematical and philosophical speculations were not historically isolated phenomena that arose completely de novo, as it were like fulgurations of genius out of a conceptually unclouded sky. They were more like nodal points in a long series of speculative endeavors that began with people like Raymon Lull in the Middle Ages, continued through René Descartes, Athanasius Kircher, Jakob Böhme, and G. W. Leibniz, that drew on the thoughts of people like Giordano Bruno and Blaise Pascal, J. D. Gergonne and Thomas Hobbes, Pierre Gregoire and Bernhard Bolzano, and in turn constituted the basis of much contemporary thought - the works of Bertrand Russell and Rudolf Carnap, Edmund Husserl and Ludwig Wittgenstein, Alonzo Church, Strawson, and Willard van Orman Quine immediately come to mind.

Remember that, and according to SEP (Stanford Encyclopedia of Philosophy), Bernard Bolzano (1781–1848) was an outstanding mathematician and one of the greatest logicians or even (as some would have it) the greatest logician who lived in the long stretch of time between G. W. Leibniz and Gottlob Frege. As far as logic is concerned, Bolzano anticipated almost exactly 100 years before Alfred Tarski and Rudolf Carnap their semantic definitions of logical truth and logical consequence; and in mathematics he is not only known for his famous Paradoxes of the Infinite, but also for certain results that have become and still are standard in textbooks of mathematics such as the Bolzano-Weierstrass theorem. Bolzano also made important contributions to other fields of knowledge in and outside of philosophy. Due to the versatility of his talents and the various fields to which he made substantial contributions, Bolzano became one of the last great polymaths in the history of ideas.

2. MANY-VALUED LOGICS AND THE LWOW-WARSAW SCHOOL

Parallel to this, there arises a new kind of thought and way of seeing must be the act of philosophizing: the Polish Lwow-Warsaw School (LWS, by acronym). This is happening like tributaries of a great river and sub-tributaries, departing from Leibniz, from masters to disciples [17, 30, 57, 61, 67, 68]. Start with the aforementioned Bernard Bolzano, which influence-much about his intellectual heir, Franz Brentano. This, in turn, greatly influence on all his subsequent students. Among these disciples of Franz Brentano will be one that particularly interested us. This was the Polish philosopher Kazimierz Twardowski, who shared many characteristics with his teacher: love for precision and clarity of ideas, charisma among those who treated him, preference for the spoken to the written word, etc… From his chair in the city of Lvov spread many of the ideas of Franz Brentano, adding their own. Its members took the logical-philosophical and mathematical studies in Poland to the forefront of global world research. It was during the "interbellum", or period between the two World Wars, i.e. ranging from 1918-1939. Then, rouse the Diaspora, after the war and by the strong communist dictatorship.

Jan Lukasiewicz introduced the three-valued logic and then generalized to the infinite-valued [36, 38]. He was the effective mentor of Alfred Tarski, whereas officially it was Stanislaw Leśniewski.

The biographers of Tarski, Anita and Solomon Feferman, state that "along with his contemporary, Kurt Gödel, he changed the face of logic in the twentieth century, especially through his work on the concept of truth and the theory of models." Tarski had gone to the US to participate in a conference when Nazi troops invaded his native Poland and could not return to it. Over time, he created in California the most powerful logical school of his time; in fact, you can consider continuing the tradition inaugurated by the LWS, outside the continent in ruins (Europe). Its `Semantic Theory of Truth´ is one of the greatest achievements of the human thinking of all time.

Many notable names were among the members of this school of logic, but could cite [57, 58, 61] to:

- Jan Lukasiewicz,
- Stanislaw Leśniewski,
- Kazimierz Ajdukiewicz,
- Tadeusz Kotarbinski,
- Mordechai Wajsberg,
- Alfred Tarski,
- Jerzy Slupecki,
- Andrzej Mostowski.

Also must be cited:

- Jan Wolenski (as vindicator of the LWS´ memory),
- Helena Rasiowa,
- Roman Sikorski,
- Zdzislaw Pawlak,
- Andrzej Skowron,
- Roman Murawski,
- etc.

Among them, one of the most interesting must be Jan Lukasiewicz, the father of many-valued logics (MVLs, by acronym). Jan Lukasiewicz began teaching at the University of Lwow, and then at Warsaw, but after World War II he had to continue at the Royal Academy of Dublin, and then at Manchester.

At first, Jan Lukasiewicz introduced the three-valued logic and then generalized to the infinite-valued. That possibility modulation can be expressed by a membership function, with values which run over all real numbers of the unit real interval, [0,1], instead of being reduced to the dichotomic {0, 1} of classical logic:

- True vs. False,
- 0 vs 1,
- White vs Black,
- etc.,

so, allowing the treatment of uncertainty and vagueness, important not only from the theoretical point of view, but from the applications.

The logical research of Lukasiewicz suffered a long slumber, until Zadeh, among others. The one who would see the potential utility in 1965, firstly obtaining a generalized version of the classical theory of sets, now denoted by FST, acronym of the so-called "Fuzzy Set Theory", and later, its application to logic, introducing the "Fuzzy Logic".

We must not forget that Zadeh, an engineer, knew Jan Lukasiewicz´s research as explained by his colleague, the brilliant American logician Stephen Cole Kleene [35].

According to this theory, we have a transfer function derived from the characteristic function usually called the "membership function", which runs from the universe of discourse, U, until the unit closed interval of reals, which is [0, 1]. Not so in the sets "classic" or "crisp sets", where the range of the function is reduced to a set consisting of only two elements, namely was the {0, 1}. Therefore, fuzzy set theory is a generalization of classical set theory [20].

We may mention the uncertainty principle of quantum physics by Werner Heisenberg. The theory of "vague sets" (today, so-called Fuzzy Sets) proceeds from the quantum physicist and German philosopher Max Black (1937), also analyzes the problem of modeling "vagueness". He differs from Russell in that he proposes that traditional logic can be used by representing vagueness at an appropriate level of detail and suggests that Russell's definition of vagueness confuses vagueness with generality. He discusses vagueness of terms or symbols by using borderline cases where it is unclear whether the term can be used to describe the case. When discussing scientific measurement he points out "the indeterminacy which is characteristic in vagueness is present also in all scientific measurement".

An idea put forward by Black is the idea of a consistency profile or curve to enable some analysis of the ambiguity of a word or symbol. To the fuzzy logic researcher of today these curves bear a strong resemblance to the membership functions of (type-1)-fuzzy sets. Also may be considered the subsequent contribution of the Polish Jan Lukasiewicz (1878-1956).

So, they must have greatly influenced Lofti A. Zadeh (b. 1921) to publish his seminal paper in the journal *Information and Control*, and three years later (since 1968), the so-called "Fuzzy Algorithm" [62-64].

In 1923, the British logician Bertrand Russell wrote that all traditional logic habitually assumes that precise symbols are being employed. It is, therefore, not applicable to this terrestrial life but only to an imagined celestial existence.

And Lofti Asker Zadeh, says that according the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.

For such reasons -during the last decades- some very powerful `Fuzzy Mathematics' has been developed, basically in Japan, but also in Europe, where these ideas came to fruition, creating a powerful technological "boom", with new techniques based on "fuzzy" concepts. This trend was particularly strong in Oriental countries, such as South Korea, China or India.

And much later these ideas, even more applications came to Western countries, both European and American, with brilliant studies both from a mathematical point of view and its philosophical implications,

as always connected therewith. Very active research groups [26], where papers on Many-Valued Logic have been published, proceed currently from good European universities, for instance of:

- Warsaw,
- Prague,
- Ostrava,
- Vienna,
- Lisbon,
- Opole,
- Barcelona,
- Madrid,
- Toulouse,
- Pamplona,
- Granada,
- etc.

Today, some emerging countries, such as Brazil (Newton Da Costa or Jean-Yves Béziau), India or Turkey, are becoming related with the investigation of all these theories and associated methods, paraconsistent logics and so one [1-9].

From the above it follows that you may need a radical rethink of our classical concepts of truth and falsehood, replacing the concept of fuzziness (vagueness or fuzziness) as a result of which the truth or falsity are only extreme cases. By fuzziness we understand the fact that a proposition may be partially true and partially false simultaneously. A person is not just tall or short, but partially may participate in both features, so that only above and below certain heights it is necessary to call upper or lower bound, while in the intermediate zone of both heights exist as a graduation which is ceasing to be high. It seems intuitively clear that the concept of fuzziness is rooted in most of our ways of thinking and speaking [27, 28]. Another separate issue is the valuation of that each individual granted such a fuzzy character (the glass half full or half empty), which depend on subjective psychological issues and are difficult to evaluate.

The fuzzy principle states that everything is a matter of degree. It will be its more famous “leitmotiv”. All propositions acquire a truth value between one (true) and zero (false), inclusive. The allocation of these extreme values will only be given in the case of logical truths or falsehoods or strong inductions: “All men are mortal” can be an example of strong induction, since there is no counterexample.

The arguments for introducing the concept of fuzziness in logic have already been exposed, but it will be necessary to examine in detail some key aspects:

a) The historical background and methodological concept.

b) The possibility of building an infinite-valued formal language, and if so, try to define their properties and laws.

c) The philosophical and practical consequences stemming from such introduction

One of the most interesting cases in the history of AI is the country of Romania [42, 44, 45]. We have the greatest landmark in the person of the mathematician Grigore Constantin Moisil (1906-1973), who introduced Computer Science in the country; after he had left a very brilliant school of researchers from Romania devoted to mathematics and AI, many of them scattered around the world by the `economic diaspora´, after the Communist period. After World War II, Grigore C. Moisil started teaching Mathematical Logic and Algebra at Iasi and Bucharest, as he understood that the new emerging field of computers would have enormous repercussions for the social fabric of society. He continued working about the ideas of Shannon on Circuits, and some Lukasiewicz´s fundamental advances on Many-Valued Logics, where the Fuzzy Logic eventually derived from.

The Lukasiewicz-Moisil Algebras (LMA) was created by G. Moisil as an algebraic counterpart for the many-valued logics of J. Lukasiewicz. They are an attempt to give semantic consistency to n-valued logics.

This theory has developed to a considerable extent both as an algebraic theory of intrinsic interest and in view of its applications to logic and switching theory.

The study of LMA was followed by G. Georgescu and A. Iorgulescu, from Bucharest; also are very relevant C. Calude, G. Paun (membrane computing), and some others, in different areas. Also worthy of mentioning is the figure of Solomon Marcus (1925-2016), an inspired disciple of G. C. Moisil, because Marcus has made great contributions to many fields of Mathematics, such as Logic, Analysis, or Computational Linguistics, of which he is one of the founders and a principal contributor [42].

Antonio Monteiro (1907-1980), mathematician born in Portuguese Angola, showed that for every monadic Boolean-algebra we can construct a 3-valued Łukasiewicz-algebra, and that any 3-valued Łukasiewicz-algebra is isomorphic to a Łukasiewicz-algebra thus derived from a monadic Boolean-algebra. Roberto Cignoli says about it that since it was shown by Halmos that monadic Boolean-algebras are the algebraic counterparts of classical first order monadic calculus, Monteiro considered that the representation of 3-valued Łukasiewicz-algebras into monadic Boolean-algebras gives a proof of the consistency of 3-valued Łukasiewicz-logicas relative to classical logic". He showed that, from the algebraic point of view, the three-valued Łukasiewicz-logic stands in the same relation to constructive logic with strong negation as classical logic does to intuitionistic logic.

Of course there is an increasing production of publications on the area. But many of the best papers on Many-Valued Logics currently come from good European Universities and very active research groups. This is possible because very remarkable researchers on MVLs (in particular on Mathematical Fuzzy Logic) have created a solid and consistent basis for these theories. Such has been the case for Petr Hájek, from the Charles University (Prague), P. Cintula, Jan Pavelka, Líbor Behounek, or Vilem Novak, from Ostrava. They have powerful research groups, with publications which are among the most internationally valued in this field [27, 43].

And they are not alone, as in France we have the important task of dissemination and investigation of D. Dubois and H. Prade, Elie Sanchez (1944-2014), or B. Bouchon-Meunier. In Germany, H.-J. Zimmerman, or S. Gottwald (1943-2015). Also we find US researchers as Z. Wang, G. Klir or R. R. Yager; in Hungary (J. Fodor…), Canada (W. Pedrycz…), UK, Pays Bas (E. Kerre, B. de Baets, G. Cooman, M. De Cock…), Italy (G. Gerla, A. Di Nola…), Austria, Argentina (R. Cignoli), Brasil, Turkey, etc.

In Poland they follow the great tradition of the LWS of logic and mathematics, and with contributions to research the uncertainty topic through the Rough Sets, by Zdzislaw Pawlak (1926-2006), and continued by Andrzej Skowron, among others [26, 61].

3. RECEPTION OF MANY-VALUED LOGICS AND FUZZY LOGIC IN THE IBERIAN PENINSULA

As remarkable precursor of Automatics, we must mention the mathematician and Spanish engineer Leonardo Torres Quevedo (1852-1936).

Also, in medieval times, Raymond Lully, and his famous book, *Ars Magna.*

One of the first Hispanic scholars giving notice of the new currents was Juan David Garcia Bacca, who in 1936 published his *Introduction to modern logic,* a work very praised by I. M. Bochenski and Heinrich Schölz.

Later, try so eminent teachers, among them Alfredo Deaño (editor of the Spanish translation of Lukasiewicz´s selected papers), Miguel Sánchez-Mazas (interpreting in the more deep sense the logico-mathematical works and ideas of Gottfried Whilhelm Leibniz, as –for instance- the known "characteristica universalis"), or Manuel Sacristán (prosecuted in `academia´ due to its Marxist point of views), all them very often clashing against a very conservative and not so good innovative ideas [52, 53].

One very good initiative has been the creation in the old mining town of Mieres, and by the Government of Asturias, named the `Research Center for Artificial Intelligence and Soft Computing´, initially around someone as well-known as Enric Trillas, who can be considered the father of the introduction of Fuzzy Logic in the Spanish University curricula. This center has attracted many of the most famous international researchers, such as well-known Japanese Professor Michio Sugeno. His topics of research are very broad working, but revolve around fuzzy methods, as well as philosophical implications these carry.

Although I have left it for last, a name should not be omitted from those that appear only from time to time in Spain. I am referring to the Father Pablo Domínguez Prieto (1966-2009), Spanish philosopher and theologian [17-19], who wrote the first major book in Spain on the LWS, from his PhD thesis, at Madrid (1993). Such work is so-called *Indeterminación y Verdad. La polivalencia lógica en la Escuela de Lwow-Warsaw (Indeterminacy and Truth)*, and was published in 1995, with a foreword by Arch. J. M. Zycinski, and showing a very strong influence by Jan Wolenski.

Pablo can be considered as one of the Spanish precursors in the study of MVLs, from the philosophical and historical point of view.

Another interesting Spanish author who has been reporting these new streams of logic is Prof. Julián Velarde, with *Polyvalent Logic*, or his book *Formal Logic*, a volume II belonging to its *History of Logic*, all of them around the University of Oviedo and its service publications, or later, to the Editorial Pentalfa. Also of great interest may be his work *Gnoseology of Fuzzy Systems*, which analyzes the deep philosophical connections of these issues [56].

New research groups have been formed in recent times [29], as the Spanish institution CSIC (Consejo Superior de Investigaciones Científicas), centered in Barcelona, led by Lluis Godo and Francesc Esteva. Or the group that belongs to the UPNA (Public University of Navarra), headed by Humberto Bustince. Or in the University of Granada (lead by Miguel Delgado Calvo-Flores).

Observe that the impulse to the study of Mathematical Logic in Spanish universities came, among others, through the aforementioned Prof. Miguel Sánchez-Mazas, and also by Prof. Manuel Garrido.

In Portugal the origins of the study of AI are linked to the names of Luis Moniz Pereira, Helder Coelho and Fernando Pereira, who in 1973 created the LNEC, within which the following year formed a division of Computer Science [27, 28].

In 1977 the programming language called DEC-10 Prolog is introduced, which Helder Coelho contributes greatly to propagate in Brazil.

In 1984 is created the Portuguese Association for AI (being APPIA, by acronym), which maintains its vitality with many publications, and also organizing congresses.

Currently, there exist in Portugal four basic areas of work, related respectively with:

- Learning;
- Knowledge Representation;
- Knowledge in general, and
- Applications.

The research on Logic and AI in Portugal maintains its vitality through good publications, and also organizing very important conferences, in Lisbon, Porto, etc.

4. FINAL NOTE

In early January, about 700 scientists specialized in technological areas signed an open letter, warning of the dangers of AI. There have been many reactions to this paper, with catastrophic scenarios described in science fiction that warn of a revolution of the machines and extinction of humanity. However, the current risks are different. For instance, the ability of processing huge amounts of data by computers, which can be released to those who are in control. Today, the main threat of artificial intelligence is a misuse of the abilities of the equipment, which can extract and analyze data in bulk.

The director of the Institute of Artificial Intelligence (IIA) of Barcelona, Ramón López de Mántaras, also adds autonomous weapons as a threat: "in order to have robots soldiers is very worrying, because for a robot it is almost impossible to distinguish between an innocent civilian and a fighter".

"Today we can ensure that none of the current robots, nor any that will be in short, medium and even long term fairly, would be out of control".

"The problem is not in technology itself, but in humanity. Is more likely to be the man with evil intentions, who may produce a very possible war between humans and machines", he qualifies.

The loss of workplaces is another hazard of AI.

According to López de Mántaras,

"So far the robots moved people from repetitive or dangerous task, but with advances in artificial intelligence begin to endanger related services sector; for example, jobs".

"Experts agree that education is the most important measure. We will look for other jobs where creativity is essential, and therefore will require investment in education to add art between engineering and mathematics".

Meanwhile, "it must be given much more importance to lifelong learning so that people can be recycled more easily and be able to change careers. At least so far, the technological changes, which have destroyed workplaces, have also created other ones instead."

ACKNOWLEDGMENT

This work was supported by the MICINN´s Research Project and by the Investigation Group of our Spanish Universidad Nacional de Educación a Distancia (UNED, by acronym) that we belong to, as a Project FFI 2011-29623, entitled "El papel de las controversias en la producción de las practicas teóricas y en el fortalecimiento de la sociedad civil" (`Polemics and Controversies´, in short expression), being former I. P. the Prof. Quintin Racionero Carmona, who recently has passed away. And now is our I. P. the Prof. Cristina de Peretti Peñaranda.

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