

# Andrzej Mostowski, a biographical note

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## Life

Andrzej Stanisław Mostowski was born in Lwów,<sup>1</sup> 1st November 1913. He died 22 August 1975 in Vancouver, British Columbia, Canada. His parents were Zofia Kramstyk and Stanisław Mostowski. In 1920 his family moved to Warsaw.

In 1938 defended his PhD thesis *O niezależności definicji skończoności w systemie logiki* (On the Independence of Finiteness Definitions in a System of Logic) at Warsaw University under supervision of Kazimierz Kuratowski and Alfred Tarski.

Just after the war he obtained habilitation degree at Jagiellonian University in Kraków for the thesis *Axiom of choice for finite sets*. Firstly he got an university position in Łódź<sup>2</sup> He moved to Warsaw in 1947, and then he created one of the most influential schools of mathematical logic in the world at Warsaw University.

## 1. Main works

Selected most important research papers:

- *Über die Unabhängigkeit des Wohlordnungssatzes vom Ordnungsprinzip*, [1];
- *On direct products of theories*, [4];
- *On a generalization of quantifiers*, [7].

The most important research and survey books:

- *Sentences Undecidable in Formalized Arithmetic: An Exposition of the Theory of Kurt Godel*, citeAM52b,

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<sup>1</sup>In this time it was Austro–Hungarian part of Poland, then Poland, and nowadays Ukraine

<sup>2</sup>In this time Warsaw was completely destroyed and many institutions were temporarily moved to Łódź.

- *Thirty years of foundational studies. Lectures on the development of mathematical logic and the study of the foundations of mathematics in 1930-1964*, citeAM66
- *Constructible sets with applications*, citeAM69.

All of the mentioned works and many others are reprinted in [10] and [11]. He also published two important monographs *Logika Matematyczna* [2] and jointly with Kazimierz Kuratowski *Teoria Mnogości* [5]. They were very influential and useful as university handbooks.

## 2. Main scientific achievements

The best known theorem proved by Andrzej Mostowski is the following:

**Theorem 1 (Mostowski collapse lemma, [3]).** *Let us assume that a binary relation  $R \subseteq A^2$  is well founded ( $\forall B \subseteq A (B \neq \emptyset \Rightarrow \exists x \in B (x \text{ is } R\text{-minimal in } B))$ ) and extensional ( $\forall x, y \in A (x = y \equiv \forall z (zRx \equiv zRy))$ ),*

*then there are unique transitive set  $Z$  ( $\forall x \in Z x \subseteq Z$ ) and a bijection  $\pi : A \rightarrow Z$  such that*

$$\forall x, y \in A (xRy \equiv \pi(x) \in \pi(y)).$$

Other broadly known ideas of Andrzej Mostowski are:

- *Kleene–Mostowski hierarchy*, classification of arithmetical sets a kind of quantifier prefixes required for defining them;
- the notion of *generalized quantifier*, formulated by him in [7].

However, some of less known results of Andrzej Mostowski are very important and influential. Let us mention two of them.

One of a few hard important<sup>3</sup> results obtained before the second world war was the following:

**Theorem 2 ([1]).** *In Zermelo–Fraenkel set theory with atoms (ZFA) Zermelo theorem, saying that every set can be well ordered, is independent of Ordering Principle, saying that every set can be linearly ordered.*

The proof of this theorem was based on the method proposed by Abraham Fraenkel. Later on Andrzej Mostowski elaborated the method (see [9]). The basic construction is nowadays called *Fraenkel–Mostowski permutation models*.

Another important work was the method of obtaining decidability results for theories of the product of two models having decidable theories, see [4]. As a corollary it gives elimination of quantifiers for arithmetic of multiplication of natural numbers. This is the first published proof of the theorem that first order arithmetic of natural numbers is decidable. The theorem is commonly attributed to Thoralf Skolem.

<sup>3</sup>Let us observe that Andrzej Mostowski was 26 years old, when the result was published.

### 3. Influence of his works

In mathematical logic ideas of Andrzej Mostowski influenced mainly foundations of set theory. He was also one of important builders of model theory. Some of his ideas are also present in recursion theory. However he was thinking rather in terms of definability than computations.

In logic in larger sense the most influential idea was undoubtedly the concept of generalized quantifiers. It is studied now not only by mathematicians, but also by philosophers, linguists and computer scientists.

### 4. Students

He promoted the following PhD students: Zofia Adamowicz, Krzysztof Apt, Maciej Bryński, Andrzej Ehrenfeucht, Andrzej Grzegorzczak, Wojciech Guzicki, Andrzej Janiczak, Stanisław Krajewski, Michał Krynicki, Moshe Machover, Mihály Makkai, Wiktor Marek, Janusz Onyszkiewicz, Helena Rasiowa, Roman Sikorski, Kazimierz Wiśniewski, Andrzej Zarach, Paweł Zbierski.

Probably, the most influential in logic were Andrzej Ehrenfeucht and two oldest of them Andrzej Grzegorzczak and Helena Rasiowa.

### 5. Last words

Andrzej Mostowski founded one of greatest schools of foundations of mathematics. How great and important he was it is proved by what happened later with his group at Warsaw University. It completely disappeared.

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