Variations on Jaśkowski's Discursive Logic

Barbara Dunin-Kęplicz, Alina Powała and Andrzej Szałas

Abstract. Stanisław Jaśkowski, in his 1948-1949 papers on propositional calculus for contradictory deductive systems, proposed discursive logic D_2. The main motivation behind D_2 is the need to properly deal with contradictions that naturally appear in many areas of philosophy and discourse. The intuitive justification of this logic reflects knowledge fusion occurring when "the theses advanced by several participants in a discourse are combined into a single system." This point of view was seminal in the mid twenty century and remains visionary nowadays.

In contemporary autonomous systems operating in dynamic, unpredictable information-rich environments, distributed reasoning routinely takes place. This explains the key role of knowledge fusion, among others, in Distributed Artificial Intelligence. Therefore, different types of modern knowledge and belief bases become primarily concerned with inconsistent or lacking information. This requirement leads to recent approaches to paraconsistent and paracomplete reasoning, where nonmonotonic techniques for disambiguating inconsistencies and completing missing knowledge can be applied.

In this chapter we remind Jaśkowski's seminal, pioneering work on paraconsistent reasoning and indicate some of its relations to contemporary research on reasoning in Distributed AI.

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1. Prelude

Stanisław Jaśkowski introduced *discursive logic* D_2 (called also *discussive logic*) in his visionary papers [24, 25] (for their English versions see [26, 27]). It has been the first formal paraconsistent logic proposed in the literature and has opened a wide

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area of paraconsistent reasoning (for surveys see [2, 4, 34]). It also inspired many researchers who published many papers focusing solely on or directly motivated by D_2 (like, e.g., [3, 8, 9, 11, 31, 39]).

In defining D_2, Jaśkowski used S5 worlds to model sets of beliefs of the discussing participants. We say that a statement is a consequence of a discussion if it follows from at least one belief set (i.e., at least one S5 world). That way different participants may express contradictory statements α and $\neg\alpha$ while the conjunction $\alpha \wedge \neg\alpha$ cannot be derived.

In the current chapter we recall D_2 and indicate its connections to contemporary research on reasoning in many subareas of artificial intelligence. In particular, we provide a new formalization of Jaśkowski ideas in terms of belief structures introduced by Dunin-Kęplicz and Szałas in [18, 19, 20]. Belief structures are built over a four-valued logic of [40] with truth values $\mathfrak{t}$ (true), $\mathfrak{f}$ (false), $\mathfrak{i}$ (inconsistent) and $\mathfrak{u}$ (unknown). This new paraconsistent and paracomplete formalization provides a shift from the deductive perspective to belief bases perspective. While, in the former, reasoning depends on deriving conclusions valid in all models of premises – in the later, one derives conclusions valid in a single model representing the current state of the world. Of course, the formalization in belief structures is not equivalent to D_2 as it is well-known that there is no characterization of S5 in any finitely-valued logic [13].

In the formalism of belief structures, belief bases are understood as sets of worlds. However, these worlds can contain contradictory claims what makes them incompatible with modal worlds. Also, there is no need to use Kripke-like accessibility relation on worlds. Instead, we focus on epistemic profiles designed for reflecting the dynamics of belief formation and revision. The concept of epistemic profile embodies an individual's (alternatively called an agent) or group of individuals reasoning capabilities encompassing techniques suitable for different aspects of activities.

Arguably, Jaśkowski with his ideas addressing paraconsistent reasoning, especially in the context of discursive logics, has been much ahead of his times. To show the bridge between D_2 and contemporary research on belief bases, argumentation, knowledge representation, artificial intelligence, autonomous systems, etc., we define a new logic D_4. While formalizing Jaśkowski's ideas behind D_2, it also enjoys the following features:

- the formalization allows to distinguish among statements supplied by different participants of discussion;
- it provides tools for both paraconsistent and paracomplete reasoning, allowing for disambiguating of inconsistencies and completing missing knowledge in a nonmonotonic manner;
- it is computationally feasible: for implementation one can use 4QL, a rule language developed in [28, 29, 37].[1]

[1] For open-source interpreters of 4QL, see `4ql.org`.

Our variations on Jaśkowski's ideas start with the current prelude. Next, in Section 2 the main theme, Jaśkowski's Discursive Logic D_2, is reminded. The "movement" (Section 3) presents the main ideas and definitions related to belief structures. Then, in Sections 4–6 three variations on D_2 are composed. The first one (Section 4) presents D_4, a new four-valued formalization of Jaśkowski's intuitions behind D_2. The second variation (Section 5) relates D_2 and D_4 to contemporary research on dialogues. The last variation (Section 6) elaborates on connections of discursive logics to selected work on argumentation. Finally, the coda (Section 7) concludes our variations.

2. Theme: Jaśkowski's Discursive Logic D_2

In his papers on D_2 [24, 25, 26, 27], Jaśkowski addressed the following problem:

> "[...] the problem of the logic of contradictory systems [inconsistent systems] is formulated here in the following manner: the task is to find a system of the sentential calculus which: 1) when applied to contradictory systems would not always entail their over-completeness; 2) would be rich enough to enable practical inference; 3) would have an intuitive justification."

For simplicity, as the underlying logic Jaśkowski has chosen propositional modal logic S5 with usual classical connectives $\neg, \wedge, \vee, \rightarrow, \equiv$ together with modalities $\Box, \Diamond$, and considered additional connectives:

- discussive implication: $p \rightarrow_d q \overset{\text{def}}{\equiv} (\Diamond p \rightarrow q)$;
- discussive equivalence: $p \equiv_d q \overset{\text{def}}{\equiv} (p \rightarrow_d q) \wedge_d (q \rightarrow_d p)$;
- discussive conjunction: $p \wedge_d q \overset{\text{def}}{\equiv} (p \wedge \Diamond q)$.

As summarized in [34],

> "we think of each participant's belief set as the set of sentences true at a world in a S5 model M. Thus, a sentence α asserted by a participant in a discourse is interpreted as "it is possible that α" ($\Diamond\alpha$)."

Let us now define the discursive consequence relation. For a similar formulation see, e.g., Example 24 of [7]. We shall need the following translation function from D_2 formulas into S5 formulas:

$$Tr(p) \overset{\text{def}}{=} p \text{ for } p \text{ being a propositional variable;}$$
$$Tr(\alpha \wedge_d \beta) \overset{\text{def}}{=} Tr(\alpha) \wedge \Diamond Tr(\beta);$$
$$Tr(\alpha \rightarrow_d \beta) \overset{\text{def}}{=} \Diamond Tr(\alpha) \rightarrow Tr(\beta);$$
$$Tr(\alpha \equiv_d \beta) \overset{\text{def}}{=} (\Diamond Tr(\alpha) \rightarrow Tr(\beta)) \wedge (\Diamond Tr(\beta) \rightarrow Tr(\alpha)).$$

We assume that Tr preserves all other connectives and, for a set of formulas F, $Tr(F) \overset{\text{def}}{=} \{Tr(\alpha) \mid \alpha \in F\}$ and $\Diamond F \overset{\text{def}}{=} \{\Diamond\alpha \mid \alpha \in F\}$.

Definition 2.1. The *discursive consequence relation*, $\Vdash_{D_2}$ is defined by:

$$F \Vdash_{D_2} \alpha \;\text{ iff }\; \Diamond Tr(F) \Vdash_{S5} \Diamond Tr(\alpha), \tag{2.1}$$

where F is a set of formulas and α is a formula. ◁

We are now in position to recall Jaśkowski's motivations concerning discussive connectives $\rightarrow_d$, $\wedge_d$ and $\equiv_d$.

First, the motivation behind $\rightarrow_d$, as stated by Jaśkowski (see [26]), is the failure of modus ponens-based reasoning when traditional implication is used:

> "If implication is interpreted so as it is done in two-valued logic, then out of the two theses one of which is $p \rightarrow q$ and thus states "it is possible that if p then q", and the other is p, and thus states "it is possible that p", it does not follow that "it is possible that q", so that the thesis q does not follow intuitively, as the rule of modus ponens requires.
> [...] This is why in the search for a "logic of discourse" the prime task is to choose such a function which, when applied to discursive theses, would play the role analogous to that which in ordinary systems is played by implication."

Indeed, from p (i.e., $\Diamond p$) together with $p \rightarrow_d q$ (i.e. $\Diamond p \rightarrow q$) we can deduce q (so $\Diamond q$, too).

The discussive conjunction and equivalence are motivated by the following important theorem (see [27]):

> "Each thesis α of the two-valued classical calculus containing no other symbols than $\rightarrow$, $\equiv$, $\vee$ or $\wedge$ is transformed into thesis of the discussive calculus D_2 by replacing in α functors $\rightarrow$ by $\rightarrow_d$, $\equiv$ by $\equiv_d$, and $\wedge$ by $\wedge_d$, respectively."

Additionally, discussive conjunction maintains the *adjunction principle* according to which $p, \neg p \models p \wedge \neg p$. Namely, for $\wedge_d$ adjunction holds [25], since from $p, \neg p$ one can deduce $p \wedge_d \neg p$.

3. Movement: Belief Bases and Belief Structures

This section is based on [18]. However, for clarity, we restrict the presentation to propositional logic. We use the classical propositional syntax but the presented semantics substantially differs from the classical one. Namely,

- truth values $\mathfrak{t}, \mathfrak{i}, \mathfrak{u}, \mathfrak{f}$ (true, inconsistent, unknown, false) are explicitly present;
- the semantics is based on sets of literals rather than on valuations of propositional variables.

This allows one to deal with the lack of information as well as inconsistencies. The underlying semantics of propositional connectives is the one of [40]. It is summarized in Table 1. Observe that definitions of $\wedge$ and $\vee$ reflect minimum and maximum w.r.t. the ordering:

$$\mathfrak{f} < \mathfrak{u} < \mathfrak{i} < \mathfrak{t}, \tag{3.1}$$

as advocated, e.g., in [10, 28, 37, 40]. Such a truth ordering appears to be natural and reflecting intuitions of the classical two-valued logic. For example, a conjunction is true if all its operands are true, etc.

TABLE 1. Truth tables for $\wedge$, $\vee$, $\rightarrow$ and $\neg$ (see [28, 29, 40]).

$\wedge$	$\mathfrak{f}$	$\mathfrak{u}$	$\mathfrak{i}$	$\mathfrak{t}$
$\mathfrak{f}$	$\mathfrak{f}$	$\mathfrak{f}$	$\mathfrak{f}$	$\mathfrak{f}$
$\mathfrak{u}$	$\mathfrak{f}$	$\mathfrak{u}$	$\mathfrak{u}$	$\mathfrak{u}$
$\mathfrak{i}$	$\mathfrak{f}$	$\mathfrak{u}$	$\mathfrak{i}$	$\mathfrak{i}$
$\mathfrak{t}$	$\mathfrak{f}$	$\mathfrak{u}$	$\mathfrak{i}$	$\mathfrak{t}$

$\vee$	$\mathfrak{f}$	$\mathfrak{u}$	$\mathfrak{i}$	$\mathfrak{t}$
$\mathfrak{f}$	$\mathfrak{f}$	$\mathfrak{u}$	$\mathfrak{i}$	$\mathfrak{t}$
$\mathfrak{u}$	$\mathfrak{u}$	$\mathfrak{u}$	$\mathfrak{i}$	$\mathfrak{t}$
$\mathfrak{i}$	$\mathfrak{i}$	$\mathfrak{i}$	$\mathfrak{i}$	$\mathfrak{t}$
$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$

$\rightarrow$	$\mathfrak{f}$	$\mathfrak{u}$	$\mathfrak{i}$	$\mathfrak{t}$
$\mathfrak{f}$	$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$
$\mathfrak{u}$	$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$	$\mathfrak{t}$
$\mathfrak{i}$	$\mathfrak{f}$	$\mathfrak{f}$	$\mathfrak{t}$	$\mathfrak{f}$
$\mathfrak{t}$	$\mathfrak{f}$	$\mathfrak{f}$	$\mathfrak{t}$	$\mathfrak{t}$

	$\neg$
$\mathfrak{f}$	$\mathfrak{t}$
$\mathfrak{u}$	$\mathfrak{u}$
$\mathfrak{i}$	$\mathfrak{i}$
$\mathfrak{t}$	$\mathfrak{f}$

A *positive literal* is a propositional variable and a *negative literal* is a negated propositional variable.

Definition 3.1. The *truth value* of a literal ℓ w.r.t. a set of literals L, denoted by $\ell(L)$, is defined as follows:

$$\ell(L) \stackrel{\text{def}}{=} \begin{cases} \mathfrak{t} & \text{if } \ell \in L \text{ and } (\neg\ell) \notin L; \\ \mathfrak{i} & \text{if } \ell \in L \text{ and } (\neg\ell) \in L; \\ \mathfrak{u} & \text{if } \ell \notin L \text{ and } (\neg\ell) \notin L; \\ \mathfrak{f} & \text{if } \ell \notin L \text{ and } (\neg\ell) \in L. \end{cases} \qquad \triangleleft$$

Definition 3.1 is extended to all propositional formulas in the standard way, using the semantics provided in Table 1.

If S is a set then by $\text{FIN}(S)$ we understand the set of all finite subsets of S. By $\mathbb{C}$ we denote the set of all finite sets of literals.

Definition 3.2. By a *belief base* we understand any finite set Δ of finite sets of literals, i.e., any finite set $\Delta \subseteq \mathbb{C}$. $\triangleleft$

Note that such belief bases can be tractably implemented using the 4QL rule language [28, 29, 37]. They serve as basis for belief structures. Indeed, constituents and consequents being basic building blocks of belief structures are, in fact, belief bases in the sense of Definition 3.2.

By *information ordering* we understand the ordering on truth values shown in Figure 1. This ordering reflects the process of gathering and fusing information. Starting from the lack of information, in the course of belief acquisition, evidence supporting or denying investigated hypotheses are collected. This finally permits one to decide about the truth value of the hypotheses.

Definition 3.3. Let Δ be a belief base and α be a formula. We define the *belief operator* by: $\text{Bel}_\Delta(\alpha) \stackrel{\text{def}}{\equiv} \text{LUB}\{\alpha(D) \mid D \in \Delta\}$, where LUB denotes the least upper bound w.r.t. the ordering shown in Figure 1. $\triangleleft$

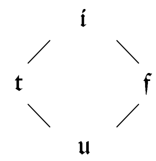

FIGURE 1. Information ordering on truth values.

For clarity let us indicate that:

$$\mathrm{Bel}_{\Delta}\big(t\big) = t \text{ when } t \in \{\mathfrak{t}, \mathfrak{i}, \mathfrak{f}, \mathfrak{u}\}. \tag{3.2}$$

Note that sets $D \in \Delta$ appearing in Definition 3.3 can be considered as four-valued worlds. Comparing to Kripke-like semantics for beliefs (see, e.g., [21]), at this point the main differences are:

- we do not require fixed, rigid structure connecting worlds via accessibility relations;
- we use four rather than two truth values.

We are now ready to define (indeterministic) belief structures, as in [18].[2] Belief structures consist of constituents and consequents: an agent starts with constituents, which are further transformed into consequents via the agent's or group's epistemic profile. While constituents contain initial, "raw" beliefs acquired by perception, expert-supplied knowledge, communication, discussion and other ways, consequents contain final, "mature" beliefs. In short, an epistemic profile encapsulates agents' or groups' reasoning capabilities, including methods of both disambiguation of inconsistencies and completing missing information.

Definition 3.4.

- By a *constituent* we understand any set $C \in \mathbb{C}$;
- by an *indeterministic epistemic profile* we understand any function $\mathcal{E}$ of the sort $\mathrm{FIN}(\mathbb{C}) \longrightarrow \mathrm{FIN}(\mathbb{C})$;
- by an *indeterministic belief structure over an indeterministic epistemic profile* $\mathcal{E}$ we mean $\mathcal{B}^{\mathcal{E}} = \langle \mathcal{C}, \mathcal{F} \rangle$, where:
  - $\mathcal{C} \subseteq \mathbb{C}$ is a nonempty set of constituents;
  - $\mathcal{F} \stackrel{\mathrm{def}}{=} \mathcal{E}(\mathcal{C})$ is the set of *consequents* of $\mathcal{B}^{\mathcal{E}}$. ◁

A formula is $\mathrm{Bel}\big(\big)$*-free* if it does not contain belief operators. Let us emphasize that $\mathrm{Bel}\big(\big)$-free formulas reflect properties of initial beliefs, being evaluated in constituents while the belief operator $\mathrm{Bel}\big(\big)$ refers to consequents, so allows us to express properties of final beliefs, as stated in the following definition.

[2]Note that epistemic profiles of [19, 20] are functions of the sort $\mathrm{FIN}(\mathbb{C}) \longrightarrow \mathbb{C}$. That is, they basically are deterministic epistemic profiles with $\mathcal{F}$ consisting of one consequent.

Definition 3.5. Let $\mathcal{B}_1^{\mathcal{E}} = \langle \mathcal{C}_1, \mathcal{F}_1 \rangle$ and $\mathcal{B}_2^{\mathcal{P}} = \langle \mathcal{C}_2, \mathcal{F}_2 \rangle$ be indeterministic belief structures. The semantics of formulas is defined by:

$$\alpha(\mathcal{B}_1^{\mathcal{E}}) \stackrel{\text{def}}{=} \begin{cases} \mathrm{Bel}_{\mathcal{C}_1}\big(\alpha\big) & \text{when } \alpha \text{ is } \mathrm{Bel}\big()\text{-free;} \\ \mathrm{Bel}_{\mathcal{F}_2}\big(\beta\big) & \text{when } \alpha \text{ is of the form } \mathrm{Bel}_{\mathcal{B}_2^{\mathcal{P}}}\big(\beta\big) \text{ and } \beta \text{ is } \mathrm{Bel}\big()\text{-free,} \end{cases}$$

where $\mathrm{Bel}_{\mathcal{C}_1,v}\big(\alpha\big)$ and $\mathrm{Bel}_{\mathcal{F}_2,v}\big(\beta\big)$ are defined in Definition 3.3.[3] ◁

The above definition can be extended for all formulas by defining the semantics of connectives as in Section 3 and nested $\mathrm{Bel}\big()$ operators starting from the innermost ones.

Recall after [18] that typical requirements as to belief operators are satisfied, where α is any formula and $\mathcal{B}^{\mathcal{E}}$ is any belief structure:[4]

$$\big(\neg\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\mathsf{f}\big)\big)\big(\mathcal{B}^{\mathcal{E}}\big) = \mathsf{t} \qquad \text{(consistency of beliefs)}$$
$$\big(\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\alpha\big) \rightarrow \mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\alpha\big)\big)\big)\big(\mathcal{B}^{\mathcal{E}}\big) = \mathsf{t} \qquad \text{(positive introspection)}$$
$$\big(\neg\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\alpha\big) \rightarrow \mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\neg\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\alpha\big)\big)\big)\big(\mathcal{B}^{\mathcal{E}}\big) = \mathsf{t} \qquad \text{(negative introspection)}$$

4. Variations Part I: D_4– a new Framework for Discursive Logics

D_2 has a potential to be extended in many directions. In particular, the following aspects can be addressed.

- A participant in a discussion should be allowed to submit inconsistent statements, as advocated, among others, in [23]. Therefore, the relevant worlds should not exclude contradictory statements, as it happens in S5, so in D_2, too.
- In contemporary systems it is often important to distinguish among statements supplied by various participants of distributed reasoning and knowledge fusion. This aspect might be essential in formulating adequate strategies of disambiguation of inconsistencies.
- As the accessible information may be incomplete as well, to reflect this property not only paraconsistent but also paracomplete reasoning is often needed.

In order to formalize Jaśkowski's intuitions behind discursive logic while addressing the above aspects, one can use the framework of belief structures. Technically speaking, beliefs are represented as sets of literals constituting paraconsistent belief bases. Epistemic profiles are represented as specific rules operating on possibly complex belief structures in order to draw individual conclusions. Discursive reasoning can be used to define epistemic profiles of individuals and groups.

In order to define a logic D_4, let us first assume that discussion participants have, as a group, an associated belief structure, say $\mathcal{B}^{\mathcal{E}}$. Since $\mathrm{Bel}\big()$ corresponds to modal $\Box$, we define $\Diamond$ as usually:

$$\Diamond\alpha \stackrel{\text{def}}{\equiv} \neg\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\neg\alpha\big), \tag{4.1}$$

[3]Note that, in the simplest case, $\mathcal{B}_1^{\mathcal{E}}$ and $\mathcal{B}_2^{\mathcal{E}}$ can be identical.

[4]Observe that the property of consistency of beliefs requires beliefs to exclude only falsity f. On the other hand, beliefs can contain contradictory claims.

and, consequently, modify translation Tr, in such a way that wherever $\Diamond$ occurs, it is replaced by $\neg\mathrm{Bel}\big(\big)\neg$. We denote this modified translation by Tr_m.

To compute the consequences according to Definition 2.1, we have to evaluate the formula $\Diamond Tr(F) \Vdash_{\mathrm{S5}} \Diamond Tr(\alpha)$. Since formulas involved are completely modalized, we use deduction theorem for S5 [43] and obtain that $\Diamond Tr(F) \Vdash_{\mathrm{S5}} \Diamond Tr(\alpha)$ is equivalent to:

$$\Vdash_{\mathrm{S5}} \Diamond Tr(F) \to \Diamond Tr(\alpha). \tag{4.2}$$

Now, rather than using S5, we use our formalization by evaluating the implication:

$$\Big(\bigwedge_{\phi\in F} \neg\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\neg Tr_m(\phi)\big)\Big) \to \neg\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}}\big(\neg Tr_m(\alpha)\big). \tag{4.3}$$

To distinguish among different discussion participants, we consider operators $\Diamond_A$, where A is a discussion participant, rather than just $\Diamond$ as in the original D_2. This is a rather immediate extension of the method outlined above. Namely, the translation Tr_m should be applicable to modal operators $\Diamond_A$, so we replace such operators by $\neg\mathrm{Bel}_{\mathcal{B}^{\mathcal{E}}_A}\big(\neg\ldots\big)$, where $\mathcal{B}^{\mathcal{E}}_A$ is a belief structure associated with participant A. Now one can use (4.3) with such modified translation Tr_m.

5. Variations Part II: Relation to Dialogue

Complex communication patterns are essential in intelligent systems. Nowadays, rather than rigid communication protocols, more relaxed communication forms are developed. Indeed, communicative actions are „actions that change your mind" [38]. Taking a commonsense reasoning perspective calls for defeasible reasoning.

Contemporary approaches to communication in intelligent systems draw upon Walton and Krabbe's semi-formal theory of dialogue [42], adapting the normative models of human communication, including paradigmatic dialogue types like inquiry, information seeking, deliberation, persuasion and negotiation. See [5, 6, 12, 21, 30, 32, 33, 35] for investigations in argumentation-based dialogue, and [42] for the definitions of dialogue types. Each model of dialogue is defined by its initial situation, the participants' individual goals, and the aim of the dialogue as a whole (see Table 2).

Complex dialogues are composed with the use of *speech acts* – the basic building blocks of communication. Contemporary understanding of speech acts comes form the works of Austin and Searle [1, 36] including the most popular taxonomy of speech acts, identifying:

- *assertives*, committing to the truth of a proposition, e.g., stating;
- *directives*, which get the hearer to do something, e.g., asking;
- *commissives*, committing the speaker to some future action, e.g., promising;
- *expressives*, expressing a psychological state, e.g., thanking;
- *declaratives*, changing reality according to the proposition e.g., baptizing

Recently we developed a paraconsistent, paracomplete, dynamic and tractable formal model of communication including:

Table 2. Types of dialogue recalled from [41].

Type of Dialogue	**Initial Situation**	**Participants' Goal**	**Goal of Dialogue**
Persuasion	Conflict of Opinions	Persuade Other Party	Resolve or Clarify Issue
Inquiry	Need to Have Proof	Find and Verify Evidence	Prove (Disprove) Hypothesis
Negotiation	Conflict of Interests	Get What You Most Want	Reasonable Settlement Both Can Live With
Information Seeking	Need Information	Acquire or Give Information	Exchange Information
Deliberation	Dilemma or Practical	Choice Coordinate Goals and Actions	Decide Best Available Course of Actions
Eristics	Personal Conflict	Verbally Hit Out at Opponent	Reveal Deeper Basis of Conflict

- a formal model of speech-acts and reasoning schemes [17, 16];
- formalization of *inquiry* as a dialogue type for *knowledge acquisition* [15];
- formalization of *persuasion* as a dialogue type for *conflict resolution* [14].

Such a model of communication can be used to enrich D_2 by developing discussion patterns and related schemes.

The nature of multi-party inquiry and persuasion dialogues resembles distributed defeasible reasoning processes, especially collective problem solving. The complex logical architecture of both dialogue types permits to associate specific belief structures with each of them. Namely, the specific rules governing each dialog type are included in the epistemic profile of a discussing group. Also specific methods for disambiguation of inconsistencies and information completion, specific for inquiry and persuasion are included in the involved epistemic profiles. Such an encapsulation of methods in epistemic profiles permits to effectively model and investigate different dialogue types indicated in Table 2. Technically, with each dialogue D, terminated or in progress, a specific epistemic profile and a belief structure $\mathcal{B}^{\mathcal{D}}$ is associated and one can use belief operators $\mathrm{Bel}_{\mathcal{B}^{\mathcal{D}}}()$ to formalize Jaśkowski's discursive connectives, as outlined in Section 4.

Using this framework, one can obtain a rich formalism, adjustable to a variety of dialogue types indicated in Table 2. Such a broader scope can still be rooted in D_2 or D_4, and deserves further investigations.[5]

6. Variations Part III: Relation to Argumentation

In realistic environments, heterogeneity of argumentation participants w.r.t. reasoning manifests itself in different conclusions drawn by participants even facing

[5]Of course, one should take into considerations rich theories developed outside of logical formalisms, in particular in the case of negotiations.

the same evidence. The notion of epistemic profile directly exposes this concept. In its abstract form, epistemic profile, being arbitrary function, conveys all reasoning capabilities of an argumentation participant. Due to this generic definition, also non-deductive reasoning methods like argumentation schemes, can be included as a part of epistemic profiles.

Argumentation schemes, originating from legal argumentation, attempt to classify different types of everyday arguments, utilizing the ideas underlying non-monotonic formalisms. Each scheme is accompanied by a set of critical questions, used to evaluate the argument. Although particular schemes may represent different types of reasoning (e.g., deduction, induction, abduction, presumption), in general they aim to model plausible, thus defeasible, reasoning.

In [16], paraconsistent argumentation schemes are modeled with the use of two dedicated sets of *premises* and *exceptions*. Intuitively, when all premises are present and none of the exceptions is present, the conclusion of the scheme can be drawn. To model such schemes, we consider three sets of ground literals: premises (P), exceptions (E) and conclusions (Con), together with a function $\mathcal{PAS}(\{P, E\}) = Con$, which represents the paraconsistent argumentation scheme. The set P contains *candidates for conclusion* of the scheme. They are obtained by means specific to every argumentation scheme. The elements of E are *triggers* that, when present, prevent drawing the respective candidate conclusion. Intuitively, a conclusion c cannot be obtained when the exceptions indicate $\neg c$. Ultimately, the conclusion of the scheme is obtained as follows. If there exists a candidate for a conclusion $c \in P$ (value of c is not $\mathfrak{u}$), check whether there exists a trigger $\neg c \in E$ blocking this candidate (value of $\neg c$ is $\mathfrak{t}$). If the trigger:

- does not exist, the candidate conclusion becomes the final scheme conclusion,
- exists, the scheme cannot be applied causing the value of $c \in Con$ to be $\mathfrak{u}$.

In short, a conclusion c is established based on the supporting arguments given by the set P (i.e., $c(P, v) \neq \mathfrak{u}$) and (lack of) rebutting triggers provided by the set E (i.e. $\neg c(E, v) \neq \mathfrak{t}$).

The definition below presents the paraconsistent argumentation scheme as a partial function: a fragment of agent's epistemic profile that expresses agent's or group's argumentative skills.

Definition 6.1. Let P and E be two constituents, representing the set of premises and exceptions, respectively, and let $\mathcal{S} = \{P, E\} \subseteq \mathbb{C}$ be a nonempty set of constituents. Then, a *paraconsistent argumentation scheme* (over $\mathcal{S}$ and Con) is a partial function: $\mathcal{PAS} : \textsc{Fin}(\mathbb{C}) \to \mathbb{C}$ such that for $Con \stackrel{\text{def}}{=} \mathcal{PAS}(\{P, E\})$ and c being a literal, we have:

$$c(Con) \stackrel{\text{def}}{=} \begin{cases} \mathfrak{t} & \text{iff} \quad c(P) = \mathfrak{t} \text{ and } \neg c(E) \neq \mathfrak{t}; \\ \mathfrak{i} & \text{iff} \quad c(P) = \mathfrak{i} \text{ and } \neg c(E) \neq \mathfrak{t}; \\ \mathfrak{u} & \text{iff} \quad c(P) = \mathfrak{u} \text{ or } \neg c(E) = \mathfrak{t}; \\ \mathfrak{f} & \text{iff} \quad c(P) = \mathfrak{f} \text{ and } \neg c(E) \neq \mathfrak{t}. \end{cases}$$

By a *belief structure associated with* $\mathcal{PAS}$ we mean $\mathcal{B}^{\mathcal{PAS}} = \langle \mathcal{S}, \{Con\} \rangle$. ◁

Note that the belief structure $\mathcal{B}^{\mathcal{PAS}}$ in Definition 6.1 is, in fact, deterministic as the set of consequents contains only *Con*. This reflects the intuition that conclusions are determined, e.g., by applying belief operators. A more comprehensive theory of argumentation and communication founded on belief structures and 4QL, TalkLog, is presented in [14, 15, 17, 16]. Observe that 4QL as the implementation tool guarantees the tractability of approach [28, 29, 37].

All and all, as in the case of dialogues, paraconsistent (and paracomplete) argumentation schemes can be viewed as a part of an agent's epistemic profile utilizing the notions of belief structures that can be directly translated into 4QL. Since with every paraconsistent argumentation schema $\mathcal{PAS}$ there is an associated belief structure $\mathcal{B}^{\mathcal{PAS}}$, one can consider belief operators $\mathrm{Bel}_{\mathcal{B}^{\mathcal{PAS}}}()$ and other operators of Jaśkowski's discursive logic, as indicated in the end of Section 6. This framework, as in the case of dialogues, opens a wide spectrum of applications of D_2 and D_4 in modeling argumentation schemes and reasoning about them.

7. Coda

Jaśkowski;s discursive logic occupies a meaningful place in philosophical logic from the moment of its inauguration. Importantly, nowadays we observe an increased demand for paraconsistent logics, which is stimulated by the needs of complex, real world applications. As Dov Gabbay [22] noticed, „New logic areas have become established and the old areas were enriched and expanded". D_2 fits in perfectly with this current trend.

As expressed in Jaśkowski's motivations behind discursive logic, inconsistency should not immediately trivialize reasoning. This approach opens up the opportunity to continue inference even when some information sources deliver contradictory information. In real-world complex applications such a situation might be common for many practical reasons. Ultimately, the inconsistencies are typically being resolved according to a chosen strategy as to timing which, again, depends on the application in question. Apparently, various forms of defeasible reasoning are applicable in this context.

In the current paper, when defining D_4 we indicate a shift from modal perspective, with reasoning over arbitrary theories, to reasoning from knowledge bases. While modeling the world and reasoning usually ends up in models of high complexity, we generally have more humble expectations from contemporary intelligent systems. We, therefore, often lean to tailor the reasoning to rule-based approaches. Long investigations on complexity of reasoning, in particular in the field of descriptive complexity, provide us with a very good picture of what is and what is not tractable and supports this shift. Therefore, a knowledge base perspective on reasoning presented in this chapter is beneficial also from the complexity point of view.

Taking into account highly complex nature of environments real-world intelligent systems are embedded in, the use of paracomplete and paraconsistent reasoning methods proves invaluable. Also within that picture, Jaśkowski's ideas are viable and inspiring.

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Barbara Dunin-Kęplicz
Institute of Informatics
University of Warsaw
Banacha 2, 02-097 Warsaw, Poland
e-mail: `keplicz@mimuw.edu.pl`

Alina Powała
(formerly Alina Strachocka)
Institute of Informatics
University of Warsaw
Banacha 2, 02-097 Warsaw, Poland
e-mail: `alina.powala@mimuw.edu.pl`

Andrzej Szałas
Institute of Informatics
University of Warsaw
Banacha 2, 02-097 Warsaw, Poland
and
Department of Computer and Information Science
Linköping University
581 83 Linköping, Sweden
e-mail: `andrzej.szalas@{mimuw.edu.pl, liu.se}`