The Absence of Multiple Universes of Discourse in the 1936 Tarski Consequence-Definition Paper

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Dedicated to Professor Roberto Torretti, philosopher of science, historian of mathematics, teacher, friend, collaborator – on his eightieth birthday.

Abstract. This paper discusses the history of the confusion and controversies over whether the definition of consequence presented in the 11-page Tarski consequence-definition paper [63] is based on a *monistic fixed-universe* framework – like *Begriffsschrift* and *Principia Mathematica*. Monistic fixed-universe frameworks, common in pre-WWII logic, keep the range of the individual variables fixed as 'the class of all individuals'. The contrary alternative is that the definition is predicated on a *pluralistic multiple-universe* framework – like the Gödel incompleteness paper [27]. A pluralistic multiple-universe framework recognizes multiple universes of discourse serving as different ranges of the individual variables in different interpretations – as in post-WWII model theory. In the early 1960s, many logicians – mistakenly, as we show – held the 'contrary alternative' that Tarski [63] had already adopted a Gödel-type, pluralistic, multiple-universe framework. We explain that Tarski had not yet shifted out of the monistic, Frege-Russell, fixed-universe paradigm. We further argue that between his *Principia*-influenced pre-WWII Warsaw period and his model-theoretic post-WWII Berkeley period, Tarski's philosophy underwent many other radical changes.

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1. Plain semi-Post algebras

In the long history of this text even what is obvious has often been overlooked. – Norman Kretzmann on a passage in the *Organon*, Buffalo (1972).

In his now-famous 11-page consequence-definition paper [63],[1] Tarski broke with various established traditions in several ways. He focused on a 'consequence' relation instead of a converse, i.e., an 'implication' relation: 'is a consequence of' replaces 'implies'. He further separated himself by taking consequence to be a relation of an individual to a 'class' instead of a relation of an individual to an individual.[2] His consequence cannot be expressed with a sentential connective. Even more, he chose his consequence relation to concern sentences of a given fully interpreted language[3] instead of 'propositional functions', uninterpreted 'formal sentences', or 'logical forms' – on one hand – or abstract 'propositions', 'judgments', 'statements', or 'thoughts' – on the other. Moreover, his consequence was metalinguistic relative to the object-language sentences it related. Thus, it could not be 'iterated'; it was impossible to say that one consequence sentence was a consequence of a class of object-language sentences – or conversely, that one object-language sentence was a consequence of a class of consequence sentences. Some of the 'paradoxes' were thus defused. For Tarski, one given sentence of a fully interpreted language is, or is not, a consequence of a given class of sentences of the same language.[4] Each language has its own consequence relation – just as in earlier Tarski works each language had its own rules of inference.

In addition for Tarski, the respective truth-values of the given sentence and those in the given class are irrelevant to determining consequence, except of course in case the given sentence is false and all those in the given class are true. For example, in some cases a given true sentence is not a consequence of a given class of true sentences; Tarski's consequence is not a material-consequence relation. And for Tarski, [sc. logical] consequence is formal in the traditional sense: if the conclusion of a given argument is a consequence of its premises, the same relation holds between the conclusion and the premises of any other argument in the same form.[5] Finally, Tarski avoided traditional logic's reference to premise-conclusion

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[1]Unless explicitly said otherwise, 'consequence' is used in a 'logical' sense so that the expression 'logical consequence' is redundant. Thus, we are not discussing the 'consequences' of events, of actions, or of inactions, etc.

[2]'Class' is the word Tarski used [63, pp. 414–419]. But, although others had distinguished 'classes' from multiplicities such as 'sets', 'aggregates', 'collections', 'extensions', and the like, no distinction is implied in [63] or elsewhere in his collection [69].

[3]One of the *History and Philosophy of Logic* referees required us to give more emphasis to this point than it had received in the submitted version. We are grateful that this needed improvement was brought to light; it helps us to highlight a major philosophical and technical change between Tarski's pre-WWII attitude and his post-WWII thinking. After the war uninterpreted constants became legitimate and were assigned an important role.

[4]Tarski did not dwell on the philosophical ramifications of this 'relevance' or 'pertinence' requirement, which might have been foreshadowed by Aristotle. But others have noted its importance (Corcoran [8]).

[5]Tarski is not as explicit as one might wish. This point is not made in any single sentence though it can be gleaned from the paragraph that begins on p. 414. Incidentally, this is the only place in [69] that uses 'form' in the required sense of logical form. See Corcoran's [14] piece 'Logical form'.

arguments: he left it entirely up to traditional logicians to discern that an argument's being valid in their sense is its conclusion being a consequence of its premises in Tarski's terminology. They were left to infer that Tarski was defining argument validity: Tarski was silent on the connection between defining consequence and defining validity. Of course, breaking with one tradition is frequently keeping with some other tradition. In a sense our paper is about whether in his consequence-definition paper he had already broken with the Frege-Russell fixed-universe paradigm.

Although we discuss the details, background, and ramifications of Tarski's definition; as stated in the abstract, our central concern is the issue of whether his definition has a *monistic* framework that keeps the range of the individual variables fixed as 'the class of all individuals', or 'the actual universe of things' as in *Principia Mathematica*, for every interpretation, or whether – to the contrary – it has a *pluralistic* framework which, like that of the Gödel incompleteness paper [27], recognizes multiple universes of discourse serving as different ranges of the individual variables in different interpretations ('sequences' or 'models').

The 1936 consequence-definition paper did not become widely available until 1956 when J. H. Woodger translated it into English. Previously it had appeared in Polish and German versions [70, p. 409]. Church [5, p. 325], who refers only to the Polish and German versions, implied that Tarski [63] presupposed a Gödel-type changeable-universe framework. In the early 1960s, after the English had been available for years, nevertheless Corcoran and many other logicians still concurred with the Church interpretation. Now to the contrary, it is widely held that Tarski had not yet fully shifted out of the monistic Frege-Russell paradigm (Mancosu [41]). We too support the monistic, one-universe interpretation of the Tarski consequence-definition paper [63]. In other words, we argue for the absence of multiple universes of discourse in the Tarski consequence-definition paper [63].

After the issue of interpreting the 1936 paper came to Corcoran's attention in 1964, he reread the paper and felt forced to recognize his mistake. From then on his understanding has been that, contrary to Church and to his own former belief, Tarski [63] presupposes a monistic *Principia*-type fixed-universe framework within which the individual variables have a range which is fixed throughout and which does not change from interpretation to interpretation. The contrast between the two approaches to explicating 'logical truth' and 'logical consequence' is explained in Chapter 4 of Quine's *Philosophy of Logic* [47], where the older fixed-universe framework is preferred.

The issue of whether Tarski's paper [63] employs a fixed or a changeable range of individual variables is entirely separate from the question of whether Tarski in 1936 or before acknowledged the fact, already noted by Aristotle in *Posterior Analytics* (Bk. A, Ch. 10, esp. 76b10 ff.) that each science has its own separate domain of investigation, sometimes called its subject matter, or its genus (e.g., Sagüillo [51, p. 268]). As Sagüillo has amply established, there are many well-known formalizations of sciences in which the individual variables range over a universe of discourse that is wider than the domain of investigation of the science.

One example is Tarski's own 1929 geometry of solids, where – as Tarski says – the term 'individual' is used in the same way as in *Principia Mathematica*. But the domain of investigation is not the range of the individual variables; rather it is 'the class of all [sc. geometrical] solids' [70, pp. 24–29].

However, the issue is related to, but not determined by, the question of whether Tarski was aware before 1936 of the De Morgan-Boole concept of *the universe of a given discourse* (Boole [3, p. 42]; Corcoran [15, p. 242], [16, p. 941]), and if aware, whether he accepted its theoretical importance in logic.

Underlying the historical and hermeneutic issue of how the 1936 paper is faithfully interpreted, there is a list of broader and more important historical and philosophical issues. Near the top of this list, we would put the question of whether, as De Morgan and Boole held, we have the capacity to discuss a limited domain of investigation without referring to things outside of the domain and in particular without referring to the entire universe of individuals. For example, is it *possible* to say that every person is mortal using a sentence such as 'for every person x, x is mortal' with the common noun 'person' indicating the range of the variable and without in any way referring to objects other than persons? Or to the contrary, in order to say that every person is mortal is it *necessary* to use a sentence such as 'for every [object] x, if x is a person, then x is mortal' with the 'universe of individuals' as the range of the variable? Do we really have to talk about *every* individual in order to make a statement about every person?[6]

The view held by Frege and Russell is that individual variables necessarily range over the entire universe of individuals and any restriction of subject is to be accomplished by means of a qualifying condition. For example, to be more explicit, on their view, to say that every person is mortal we would be required to use a sentence such as 'for every individual x, if x is a person, then x is mortal', where the common noun 'individual' indicates that the range of the variable is taken to be the class of all individuals. In practice, the quantifier phrase 'for every individual x' is often written elliptically as 'for every x' with the *range indicator* 'individual' to be 'understood'. Moreover, the entire quantifier phrase 'for every individual x as Tarski [66, p. 7] says, '[...] is often omitted and has to be inserted mentally': 'if x is a person, then x is mortal' is used to express the proposition 'for every individual x, if x is a person, then x is mortal'.[7] On this view, Aristotle's universal affirmative propositions cannot be expressed in a logically perfect language without using the truth-functional connective 'if-then' and using a variable that ranges over the entire universe of individuals.

Boole espoused the *Principle of Wholistic Reference* (PWR): every proposition refers to *its* entire universe of discourse, regardless of how limited the number of objects explicitly referred to by its non-logical concepts Corcoran [20]). Although within a given discourse, it is impossible to refer to objects outside of the

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[6]According to Leonard Jacuzzo, whose 2005 dissertation research involved comparisons of dozens of introductory logic texts, most books he studied teach the affirmative answer to this question (personal communication). Our own less extensive experience confirms his sad finding.

[7]For more on these points see Gupta [31]; Tarski [66, Section 3]; Corcoran [17].

universe of *that* discourse, as said, there are formalizations of sciences in which the individual variables range over a universe of discourse that is wider than the domain of investigation of the science. However, frequently but – as already said – not always, the universe of discourse of a proposition is not wider; it coincides with the domain of investigation of the science in which it occurs (Sagüiillo [51]; Corcoran [15, 16]). In such cases, there need not be two different non-logical constants for the two different but coextensive concepts. But when there are two terms, the formalization often contains a 'theorem' to the effect that every object in the universe of discourse is in the domain of investigation. Tarski [61, pp. 310–311] mentions a geometry containing a sentence 'which asserts that every *individual* [sc. in the universe of discourse] is a *point* [sc. member of the domain of investigation]' – emphasis added. Thus, the universe of discourse coincides with the domain of investigation. He also mentions an 'axiom system of arithmetic' that 'contains a sentence to the effect that every *individual* [sc. in the universe of discourse] is a *number* [sc. member of the domain of investigation]' – emphasis added.

Frege and Russell carried Boole's PWR one step further: by putting all of the domains into one all-encompassing cosmic universe of individuals over which every individual variable ranged. They and many of their followers replaced PWR by the *Principle of Cosmic Reference* (PCR) – although they did not, indeed could not, put it this way. The PCR is that every proposition refers to the entire, cosmic universe of all individuals. The hermeneutic question mentioned above is whether Tarski [63] subscribed to the Principle of Cosmic Reference.[8] Our opinion is that he did: in some passages he seems to think that the individual variables range over 'all possible objects' [63, p. 416]. He never made use of the De Morgan-Boole concept of universe of discourse, i.e., discourse universes, in the 400-odd pages of *Logic, Semantics, Metamathematics.* The expression 'universe of discourse' occurs there about a dozen times: never in the consequence-definition paper, never in the plural, and never in the sense of the range of the individual variables of the object language. Sometimes, it is used for the domain of investigation of a science [70, p. 28, pp. 135–140] and sometimes it is used for the carrier[9] of a structure such as a

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[8]Space limitations preclude discussion of how Frege arrived at PCR (or a similar monistic universal variable-range view), why it was so widely adopted, how its conflict with pluralistic views escaped notice for so long, and when its soundness came under scrutiny.

[9]Today the set of elements of a Boolean algebra, or of any other algebraic structure such as a group or a ring, is called its *carrier.* The maximal element of the Boolean algebra is usually denoted by the digit '1' and is called its *unity.* If a given Boolean algebra is 'formalized' using a first-order language whose individual variables range over the carrier, then the carrier is the universe of discourse of the 'theory of the algebra'. But this can distort Boole's [3] viewpoint. Take Boole's 'universe of men (sc. humans)'. Boole used '1' to denote this class and '0' to denote the null class: two elements of the carrier of the corresponding Boolean algebra. In such cases, where a Boolean algebra of classes is under discussion, there are two things competing for the names 'universe', 'universe of discourse', 'universal class' and the like: the carrier and the carrier's unity. The carrier is often the powerset of the carrier's unity. In such cases, the carrier's unity is the union of the carrier. The Boolean tradition would incline towards using such terms for the unity. The modern abstract-algebra viewpoint that abstracts from the nature of the elements of an abstract algebra would incline toward using such terms for the carrier. Tarski called the

Boolean algebra [62, pp. 320, 322, 335, 347, 350, and 373]. Tarski's use of 'universe of discourse' is discussed further below.

Even in the late 1970s and early 1980s, the issue of single versus multiple universes in Tarski [63] was still completely (and wrongly) settled in favor of the pluralistic interpretation in the minds of all logicians Corcoran was in contact with and who cared about it, except for the few he succeeded in convincing otherwise.[10] In fact, as far as we know, there was little or no discussion of this point until Corcoran brought it up in the winter of 1964 or the spring of 1965.

2. Corcoran's awareness of the issue in the 1960s and 1970s

> People who comb or concoct obscure sources in order to support usually lame or self-serving priority claims concerning well-known discoveries, theorems, 'theses', or conjectures have their priorities reversed. It is not the first person who discovers something that counts; it is the last person – the person who discovers it so that it never needs rediscovery. – Peter Freyd, 1968 Philadelphia Logic Colloquium.

For years, without having studied Tarski [63] very carefully, Corcoran was under the impression that it contained the pluralistic, multiple-universe conception of consequence, probably to some extent on the strength of Church's assertion [5, p. 325, fn 533]. In the fall of 1964, Corcoran reread the paper carefully and was surprised that his expectations were not fulfilled: he found that Tarski had a monistic fixed-universe framework. Still, Corcoran was not very interested in the issue; he thought that Church's mistake must have been caught by many others. He let the issue drop.

Corcoran traces the awakening of his interest in the issue to a conversation that semester with William Craig[11] in Craig's Dwinelle Hall office on the Berkeley campus of the University of California late one afternoon. He recalled casually

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carrier or the set of elements of a given Boolean algebra its universe of discourse on the first page of the article beginning on p. 347 of [70].

[10]To the best of Corcoran's knowledge, there is no chance that any unpublished writing by Tarski, either a passage in the Tarski-Corcoran correspondence (preserved at the Bancroft Library at U.C. Berkeley), or anywhere else, would suggest that Tarski [63] conceives of various models having various universes of discourse. In fact, according to Paolo Mancosu, evidence to the contrary is to be found not only in the Tarski-Corcoran correspondence (Mancosu [39, p. 451]) but also in Tarski's unpublished 1940 lecture 'On the completeness and categoricity of deductive systems' also in the Bancroft Library (Mancosu [41, pp. 754–756]).

[11]Even at that time, William Craig was a distinguished mathematical logician, former doctoral student of Quine at Harvard, Full Professor of Philosophy at Berkeley. He was a member of the UC Berkeley Group for Logic and Methodology, in which Tarski was still active. He was internationally known for what was then called the Craig Interpolation Lemma. The Craig Interpolation Theorem, as it is sometimes known today, is 'one of the basic results of the theory of models, almost on a par with, say, the compactness theorem' (Boolos et al. [4, p. 260]). An entire chapter of [4] is devoted to this theorem, which is to be distinguished from a less deep but equally famous result then called Craig's theorem, now sometimes the Craig Axiomatizability Lemma or the Craig Reaxiomatization Lemma (Boolos et al. [4, p. 198]).

mentioning to Craig that Church [5, p. 325, fn 533] was not entirely accurate in crediting Tarski [63] with explication of the concept of logical consequence Church used and that had been used by the American Postulate Theorists[12] including Church's teacher Oswald Veblen. The discrepancy was that the Church concept, contrary to Tarski's, involved changing universes of discourse. Corcoran was under the impression that it was common knowledge that the Tarski [63] definition did not Vary universes'. Corcoran was surprised that Craig had not heard of this rare inaccuracy in Church [5] and even more surprised to learn that Craig thought that Corcoran must be wrong.

Corcoran remembers excusing himself for a moment, going next-door to his office, and returning with a copy of *Logic, Semantics, Metamathematics*, which he handed to Craig asking him to show where Tarski says that different interpretations can have different universes of discourse, i.e., that the individual variables can have different ranges relative to different interpretations. As befits his dignity, Craig asked permission to read at his leisure and to continue the discussion the next day. The next day he came to Corcoran's office reporting he had reread the entire article and that, to his surprise, he found no mention of alternative universes of discourse in Tarski [63].

What Tarski [63] presents is a kind of updated and transformed version of the Russellian formal implication. In Russell's sense, *one* proposition *formally implies* a *second* if and only if the 'generalized conditional' of the former with the latter is true. By the generalized conditional of one proposition with another is meant the universal closure of the result of replacing all of the non-logical constants in the conditional by suitable variables (Russell [49, pp. 5, 11, 14, and 36–41]).[13] To use an adaptation of one of Russell's examples, the single proposition 'Socrates is human and everything which is human is mortal' formally implies the single proposition 'Socrates is mortal' if and only if the following single proposition is true: 'Given any individual x, any property P, and any property Q, if x is P and everything which is P is Q, then x is Q'. There is no place here for changing universes.[14]

As already noted, one of Tarski's innovations was his broadening the situation by taking the 'implicant' to be a class of propositions (interpreted sentences),

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[12]For more on the American Postulate Theorists see Scanlan's article 'Who were the American Postulate Theorists?' [53] and for the influence they had on Tarski see the same author's 'American Postulate Theorists and Alfred Tarski' [54].

[13]We never use the expression 'formally implies' in Russell's sense without explicitly adding 'in Russell's sense' or an equivalent. In fact, as Nabrasa pointed out, few if any logicians do either. He reminded us that the fact that a person defines an expression in a certain sense, in and of itself, is no evidence that the person uses the expression in that sense (Frango Nabrasa, personal communication).

[14]This is yet another example of Russell's habit of using previously established terminology in a sense never before employed and without explaining or even alluding to the previous senses. His friend and colleague G. E. Moore criticized him for this in connection with using 'implies' in the sense of the truth-functional conditional. Previously no traditional logician would have said that 'Some animal is not a dog' logically implies 'Every oak is a tree'.

possibly infinite, and thereby avoiding the conditional altogether in favor of a metalinguistic condition.

A person scrutinizing [63] for signs of interpretations having different variable ranges might first look for the words 'domain' and 'universe'; neither of them occur there. Moreover, the first full sentence on p. 416 criticizes the no-expressible-countermodels definition[15] on the ground that it implies 'the designations of all possible objects occurred in the language in question', suggesting that he is thinking of languages whose individual variables are unrestricted in range, i.e., which have the cosmic universe of individuals as their range.[16] The fact that Corcoran's observation about the Church inaccuracy was a surprise to Craig highlighted the issue in Corcoran's mind. He recalls resolving to say something about it in print when appropriate opportunities arose.

Corcoran recalls discussing this with some of his friends, students, and teaching assistants then and over the next couple of years. He remembers Robert Barnes, Herbert Bohnert, Oswaldo Chateaubriand, William Frank, Edward Keenan, Ray Lucas, John Pollock, David Sherr, George Weaver, but there were probably some others. At the time he was in regular correspondence with Lucas (then at the University of Wisconsin) and with Bohnert (then at IBM Research). Chateaubriand, Pollock, and Barnes were in his 1964 'Philosophy of Mathematics' seminar. Frank, Keenan, Sherr, and Weaver attended Corcoran's 1966 graduate course 'Mathematical Logic', which had Church [5] as its only required text. In the late 1960s Corcoran was in constant contact with his friend and colleague Henry (or Henryk) Hiż, whose 'Reexamination of Tarski's semantics' [34] did not even touch this issue.[17]

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[15]See p. xxii of Corcoran's Editor's Introduction to Tarski [70]. A no-expressible-countermodels definition of consequence defines a premise-conclusion argument expressed in a given language to be valid if no argument in the same form expressible in the same language has all true premises and false conclusion. Such a definition is featured in Quine's *Philosophy of Logic* [47], where great care is taken to ensure that the language has the required 'richness', to use Tarski's expression [64, p. 416]. Even today, some authors claim, of course without giving any justification, that the no-expressible-countermodels conception is *the classical definition of consequence*. E.g. in [4, p. 101] where 'argument' is elliptical for 'expressed argument', we find: 'Logic teaches that the premises [...] (*logically*) *imply* or have as a (*logical*) *consequence* the conclusion [...], because [*sic*] in any argument of the same form, if the premises are true, then the conclusion is true'.

[16]Tarski should not be construed as referring to possible objects as opposed to actual objects. By '*possible objects*' Tarski means 'objects'; the use of the modal adjective 'possible' is entirely empty – it is what is sometimes called redundant rhetoric, filler, or expletive. Tarski's usage of modal words is almost always, if not absolutely always, expletive, like putting 'absolutely' before 'always', 'entirely' before 'empty', 'no matter how small' after 'every real number' or 'if any' after 'all odd perfect numbers'. See Corcoran [18] and [19, p. 266]. Incidentally, Tarski may not be speaking very strictly in saying that designations of *all* objects are needed.

[17]Henry Hiż had been a fellow Quine PhD student at Harvard with William Craig and Robert McNaughton, supervisor of Corcoran's 1963 dissertation. McNaughton introduced Corcoran to Hiż in 1961 or 1962 and to Craig in 1963. Hiż helped with the second edition of *Logic, Semantics, Metamathematics* and with the Editor's Introduction [70, pp. viii, xxvii].

Corcoran also mentioned Tarski's fixed-universe viewpoint, as it has come to be known,[18] in lectures at Georgia, Michigan, Pennsylvania, New York, Ontario, Quebec and Mexico over the next few years in connection with his work on argumentation in philosophy of logic and with his work on Aristotle. From 1971 to 1973, he circulated a series of typescripts that appeared as Corcoran [11] containing the following passage on pp. 126 and 127, bracketed matter added.

> Church [5, p. 325] attributes this [concept] [...] of logical consequence to Tarski [70, pp. 409–420], but Tarski's notion of true interpretation (model) seems too narrow [...] in that no mention of alternative universes of discourse is made or implied. In fact, the limited Tarskian notion seems to have been already known by Lewis and Langford [37, p. 342], to whom [...] I am indebted for the terms 'interpretation' and 'true interpretation', which seem heuristically superior to the Tarskian terms 'sequence' and 'model', the latter of which has engendered category mistakes – a 'model of a set of sentences' in the Tarskian sense is by no means a model, in any ordinary sense, of a set of sentences.

These points had been made in print earlier in 'Conceptual Structure of Classical Logic', (Corcoran [8, p. 43] quoted in Sagüillo [50, p. 238]), a paper written in 1969, and in 'A Mathematical Model of Aristotle's Syllogistic' (Corcoran [9]). Both papers were published in main-stream international journals with large circulations. To the best of our knowledge these points were never disputed by anyone at the time, or even in the 1980s. Scanlan and Shapiro [55, pp. 149–150] mention related events.

3. The place of the monistic-pluralistic distinction

> Here and elsewhere we shall not obtain the best insight into things until we actually see them growing from the beginning. – Aristotle, *Politics*.

It would be interesting to know when the explicit observation that Tarski [63] holds to a monistic and not to a pluralistic multiple-universe viewpoint was first made in print. More generally, we can wonder when, after say 1972, it was first noted that Tarski [63] employed a fixed-universe framework. In the 1972 article Corcoran had more pressing issues to deal with and accordingly understated his observations[19] about [63]: 'There is room to doubt whether Tarski (1936) permits changes in domain (universe of discourse)' (Corcoran [8, p. 43]). The question is not one of priority: the issue is so obvious that no one would want to claim credit for it. The question is when logicians came to feel that this obvious point needed to be made. Our evidence seems to indicate that Chapter 8 of the 1990 Etchemendy book made this explicit in reconsidering the status of pure cardinality

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[18]See Sagüillo [52], but compare with Sagüillo [50, 51].

[19]We should also point out that Corcoran is also to blame for a related misinterpretation of Tarski [63]. He said that Tarski substituted new non-logical constants in the manner of the American Postulate Theorists. Of course, Tarski substituted variables like Russell [49], not constants.

sentences. Likewise, Chapter 2 of Simons [58] brought the issue up in reconsidering the criterion for logicality sketched in the still monistic setting of Tarski [73].

It would also be interesting to know when the true statement that Tarski [63] holds a monistic viewpoint was first explicitly *denied*. In our opinion, Church [5] makes no such denial. In fact, he gives no indication that he specifically considered the issue. He does not quote one sentence from the Tarski paper. It is likely that he did not deliberate on it and it seems certain that he would have retracted his statement given the chance. Our evidence seems to indicate that Gómez-Torrente [29] was in fact the first to explicitly *deny* that Tarski held a monistic view and to *claim* that Tarski held a pluralistic view in the 1936 consequence-definition paper (cf. Mancosu [40, pp. 463–468], and [41, pp. 751–752]).

As mentioned, Tarski published two versions of the consequence-definition paper in the year 1936, one in German and one in Polish.[20] The above remarks apply directly to the 1956 English translation by Woodger of the 1936 German form of the Tarski consequence definition paper. From the 1960s though the 1990s, Corcoran had no motivation to consider the 1936 Polish form. In the intervening period Magda Stroińska and David Hitchcock of McMaster University have translated the Polish form into English and they have written extensive commentary accompanying their translation (Hitchcock and Stroińska [33]).

In comparing the 1936 Tarski conception of consequence as it appears in the Polish paper with modern conceptions, they wrote [33, p. 167]: 'Tarski [. . .] worked with formalized languages, in which [. . .] the domain is fixed.'

It is hard to believe that anyone who had read Tarski [63] and who was versed in the history of modern logic going back to Frege and Whitehead-Russell could find 'variable universes' in it. Most authors we know of who address the issue agree that the pluralistic multiple-universe viewpoint is nowhere to be found in Tarski [63]. For example, in his 1992 *Journal of Symbolic Logic* review of John

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[20]The following background has been generously supplied by David Hitchcock (personal communication): 'Tarski wrote the paper in 1935. He delivered the German version at a conference in Paris in September 1935, and appears to have left a copy of the paper with the conference organizers for publication in the conference proceedings, which came out in 1936. The Polish version appeared in the first (January) issue of the 1936 volume of *Przegląd filozoficzny* (Polish for 'Philosophical Review', the leading Polish philosophy journal), and so must have been submitted by the end of 1935, allowing time for typesetting and correcting proofs. Thus, Tarski wrote the paper no later than 1935. It is likely that he did not write it earlier than 1935, since Carnap reports in his autobiography that Tarski visited Vienna in June 1935 and that Carnap persuaded Tarski at that time to present his ideas on semantics at the September 1935 conference. Tarski had just finished translating his truth monograph into German (at the end of the historical note which Tarski added to his German translation of his truth monograph, there appears in Latin in italics, centered two lines below the end of the text the sentence

"Nachwort" allatum est die 13. Aprilis 1935.

– i.e. The 'afterword' was produced on the 13 April 1935.). The other paper that Tarski presented at the Paris conference is a kind of summary of the ideas in semantics in that work. Both papers in fact presuppose the concepts of the truth monograph.' For further information see Hitchcock-Stroińska [33, especially, pp. 155-158].

Etchemendy's *The Concept of Logical Consequence* [25], Vann McGee wrote [42, p. 254]:

> Etchemendy emphasizes the divergence between Tarski's [1936] analysis of logical truth and what we may call the modified Tarski thesis, which identifies logical truth with truth in every model. In particular, Tarski's original analysis makes no provision for the special role of the universe of discourse [. . .]. It is unfortunate that Etchemendy focuses all his fire upon the original [1936] Tarski thesis, for it is the modified thesis which Tarski himself employed in his mature work that has won nearly universal acceptance.

By omitting acknowledgement of the De Morgan-Boole theory of discourse universes, Frege – and then Russell – might have virtually ruled it out for their more dedicated disciples, as is especially clear in the writings of van Heijenoort, Chateaubriand [6, pp. 174f], and others. In 1936, Tarski had not yet developed many of the aspects of his post-WWII, Berkeley-period, multiple-universe view that *rejected* features of the Frege-Russell philosophy, e.g., higher-order logic. As indicated in Sagüillo [50, p. 233], our present evidence suggests that Tarski's earliest explicit written endorsement of the pluralistic viewpoint is in his 1953 undecidability paper. However, according to Tarski's student Roger Maddux (personal communication), the *locus classicus* most often referred to in this connection is the 1957 Tarski-Vaught paper.

It is not the case, however, that Tarski [63] explicitly *rejected* the multiple-universe viewpoint. In fact, it shows no awareness of the issue. As said, the expression 'universe of discourse' does occur several times in Tarski [69]: but never in the consequence-definition paper, never in the plural, and never in a relevant sense. After discussing formalization of 'the arithmetic of real numbers' wherein the first-order variables range over Rl the set of real numbers, he 'generalized' his remarks by taking an arbitrary set instead. He wrote [69, p. 135]: 'The set Rl is now replaced by an arbitrary set V (the so-called universe of discourse or universal set)'. There is another passage in the 400-odd page Tarski [69] – footnote 2 of p. 310 of Article X (Tarski [61]) – where Tarski implicitly recognizes the possibility of a formalized science whose domain is a proper subclass of what he calls 'the class of all individuals'. But, the expression 'universe of discourse' does not occur in this article and there is no special symbol for arbitrary universes of discourse. In fact, the symbol V is used for 'the class of all individuals'. Moreover, the first sentence of the footnote implies that Tarski thought it was 'customary' to discuss categoricity only in contexts where the individual variables were considered to range over all individuals and not merely over the genus or domain of objects relevant to the science being formalized.

We are not aware of any hint in any of Tarski's pre-WWII papers of appreciation of the De Morgan-Boole concept of the universe *of a discourse*. He never mentioned the fact that in a discourse the participants agree tacitly or explicitly to limit the subject-matter of that discourse, i.e., in modern terms, to restrict the

range of the individual variables (as opposed to affixing restrictive relative clauses to the universal sentences leaving the variables to have unrestricted range).[21] It is true that Tarski uses the expression 'universe of discourse', e.g., in the 1935 Boolean algebras paper [62] written about the same time. However, here the 'universes of discourse', indicated by a non-logical constant B used in restrictive relative clauses, are the carriers of the algebras, not the range of the individual variables. On p. 199 of the 1933 truth-definition paper, Tarski shows acquaintance with a relativized notion of truth when he said the following.

> In the investigations which are in progress at the present day in the methodology of the deductive sciences (in particular in the work of the Göttingen school grouped around Hilbert) another concept of relative character plays a much greater part than the absolute concept of truth and includes it as a special case. This is the concept of *correct or true sentence in an individual domain a.*

Tarski never makes the point (made repeatedly by De Morgan and by Boole) that each discourse[22] or discussion has its limited 'universe' as its ultimate subject matter, a point that was central to Boole's theory of propositions (Corcoran [18] and [19, p. 275]). Nevertheless, by his repeated use of 'domain-dedicated' variables, Tarski's work exemplified Boole's theory that different discourses can have different domains and that it is not necessary to make statements about *every* thing. Among the domain-dedicated first-order, individual variables used in Tarski's pre-WWII papers, we find sentential variables, string variables, and number variables not to mention the first-order object languages whose variables range over a universe of 'classes'. The most prominent of the latter, of course, is the object language whose truths are defined in the 1933 truth-definition paper [70, pp. 168ff.].

A no-countermodel view of logical consequence is implicit in the classic Padoa paper [43] on definitional and implicational independence. Some readers might be inclined to say that Padoa actually subscribed to the no-countermodels view of

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[21]The De Morgan and Boole work discovered the concept of the universe *of a discourse* in the conceptual framework of the mathematics and science of their time. The role of universes of discourse persisted into the conceptual framework of Tarski's time. Tarski's work shows this role but it does not show awareness of that role nor does it show any appreciation of the De Morgan-Boole achievement. The fact that blood circulated in Plato's veins is no reason to credit him with discovering or knowing of blood circulation.

[22]The wisdom of Boole's choice of the word 'discourse' for a certain sort of extended exposition or discussion may be questioned and it is open to doubt whether the word had ever been used in his precise sense before. Others may have observed how much Boole enriched the English language by coining the phrase 'universe of discourse', but no one seems to have suggested that his use of 'discourse' may have been equally creative and meritorious. By the way, the word 'discourse' is more often used for a stretch of speech involving typically more than one sentence-like expression, e.g., a paragraph or an argumentation. It was used in this sense in Zellig Harris's Discourse Analysis Project, which is discussed in Corcoran's paper 'Discourse grammars and the structure of mathematical reasoning, Part I: Mathematical reasoning and the stratification of language' [7]. See also Corcoran [12].

consequence. It is tempting to speculate that he would have stated it clearly except that he had not distinguished – on one hand – the *epistemic* relation of 'is-a-deduction-from' (intrinsically connected with deduction, the human capacity to reason logically) and – on the other – the *ontic* relation 'is-a-consequence-of' (extrinsically serving as the objective ground or standard of deduction). This distinction is not made despite the twin facts that Padoa refers to deduction as a human activity and that he takes the existence of a countermodel to be entirely objective, not dependent on human capabilities. The distinction is overlooked because a kind of absolute soundness and a kind of absolute completeness are presupposed [43, pp. 122–123]. Padoa's explicit statements imply, using the vocabulary of the English translation by van Heijenoort, that in order for one given proposition to be a logical consequence of a given set of propositions it is necessary and sufficient for no interpretation satisfying the set to satisfy the negation of the given proposition. Tarski was well acquainted with Padoa's thought. Moreover, although Padoa mentions individuals and variables, he never mentions change in the range of the individual variables. To all appearances, Padoa holds to a monistic fixed-universe viewpoint.

The topic of precursors of Tarski's 1936 consequence definition points to a remarkable difference between his attitude toward precursors taken in the 1936 consequence-definition paper and that taken by him in the earlier and more well-known 1933 truth-definition paper. In the earlier paper, Tarski emphasized an 'essential' similarity – perhaps identity – between the conception of truth Tarski characterized and that characterized in Aristotle's truth-definition. He quoted Aristotle: To say of what is that it is not, or what is not that it is, is false, while to say of what is that it is, or of what is not that it is that it is not, is true' [69, p. 155]. In the later paper, there is no reference to Aristotle; no comparison is made between Tarski's relation of consequence and anything Aristotle says about 'syllogistic consequence': the relation of the conclusion of a syllogism and its premises. There are two interesting points. First, in a course Tarski probably attended, Tarski's teacher Jan Łukasiewicz interpreted the syllogistic consequence to be something very close to a special case of Russellian formal implication (Łukasiewicz [38, pp. 103–112, especially p. 112]). Second, in *Prior Analytics* there is no succinct passage characterizing syllogistic consequence; there is nothing analogous to the pithy passage in *Metaphysics* characterizing truth. In fact, Aristotle seems to have deliberately avoided any verbal characterization of syllogistic consequence (Corcoran [21, pp. 151–153]).

4. The origin of the modern form of the monistic framework

> Every science which is ratiocinative or at all involves reasoning deals with causes and principles, exact or indeterminate; but all these sciences mark off some particular being – some genus, and inquire into this, but not into being simply or *qua* being. – Aristotle, *Metaphysics*, 1025b5-10.

Frege, in apparent willful ignorance of facts made clear by De Morgan and disseminated by Boole (Corcoran [16, 18, 19]), created the fictional framework of monistic fixed-universe logic. Goldfarb [28, p. 351] says that for Frege as well as for Russell the ranges of the variables 'are fixed in advance once and for all' and the 'universe of discourse is always the universe, appropriately striated'. Peano (e.g., [44]) and then Whitehead-Russell (1910) follow Frege in this regard. They also follow Frege in showing no awareness of an alternative – even though all four of them repeatedly refer to Boole 1854, where we find the shift from – at first – using ' 1' for *the* universe to – later – using it as an indexical[23] that denotes in a given discourse the universe *of that discourse* (Boole [3, p. 54], Corcoran [18, p. 254]). To the best of our knowledge, the monistic fixed-universe viewpoint has never been accepted by any logicians who knew of the alternatives, except followers of Frege such as Jean van Heijenoort, Willard Quine [47], and those who find Quine's brand of wholistic naturalism attractive. Even the post-WWII, Berkeley-period, Tarski, to his credit, adopted the multiple-universe viewpoint – although the issue was never clear to him.

In pre-WWII logic and foundations, there were two philosophies living in tense but peaceful co-existence. On one hand, there were the *monists* who recognized one single fixed universe, who worked in a framework in which the universe of discourse, or range of individual variables, remained fixed as *the* class of individuals, i.e., who subscribed to the fixed-universe viewpoint. Many of the monists pursued what van Heijenoort has somewhat mysteriously called 'logic as language'. A few of them, also somewhat mysteriously, regarded metalanguage statements as a kind of inspired and revealing incoherence, as literally meaningless, or studiously avoided saying anything about their own metalanguage, which belonged to a different discourse having a different range for its individual variables. They included Boole (at least as late as 1848), the early Frege, Peano, Padoa, Russell (in many of his writings), the early Wittgenstein, Lewis, Carnap, pre-WWII Tarski, and others. As is to be expected, the monists misunderstood, ignored, or even denigrated the De Morgan-Boole doctrine of universe of discourse. For example, Lewis-Langford [37] uses the expression 'universe of discourse' twice in its 500-odd pages. Once it is used on p. 28 in connection with the unity in the Boole-Schroeder algebra (not the carrier) and once on p. 353, *in quotes* preceded by 'as it were', in connection with interpreting a monadic letter used in restrictive relative clauses – not for the range of the individual variables (which is never changed).

In addition, monists tended to be reductionists who were not comfortable with the idea of a plurality of separate ontologically independent mathematical sciences. Many of them stretched reductionism to its logicist extreme of 'reducing' all mathematical sciences to logic. In some cases this meant exchanging an ontology of numbers, points, lines, solids, classes, vectors, etc. for an ontology

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[23]By an indexical we mean a word such as 'I', 'you', 'here', 'this', and 'now' whose denotation changes according to the context of the speech act it is used in. See Corcoran [20, pp. 159–160] for a discussion of Boole's innovations involving the symbol '1'.

that restricted literal existence to the individual entities over which the individual variables ranged, entities of the most unquestionably substantial – yet abstract – and thus the most mysterious status, knowledge of which was an article of faith. In this framework, it was natural to accept what has unfortunately come to be known as 'axioms of infinity', albeit with bad conscience in some cases. On the other hand, there were the pluralists who recognized with Boole a plurality of universes of discourse, who tended to construct formalizations of sciences having in each case its universe of discourse identified with the range of its individual variables, and who tended to avoid having a variable range over all individuals without exception. These thinkers tended to accept with Aristotle a plurality of ontologically independent mathematical sciences. Among those showing pluralistic tendencies, we find Dedekind, Poincaré, Hilbert, Veblen, Huntington, Gödel, and even Church – somewhat surprisingly given his deep appreciation of many aspects of monistic thinking in the work of Frege and Russell.[24] Hilbert's famous *Foundations of Geometry* [32], which was probably read by every logician that came later, was a stunning endorsement of pluralistic thinking: it took geometry to be an autonomous science and it had three sorts of domain-dedicated individual variables – one sort ranging over points, one ranging over lines, and one ranging over planes. Moreover, from Hilbert's consistency and independence results it is clear that his work presupposes a no-countermodels conception of consequence based on a pluralistic multiple-universe framework.

Within a monistic framework one natural way of *construing* logical consequence is a no-countermodels fixed-universe concept, one explication of which is given in the 1936 Tarski consequence-definition paper. Within a pluralistic framework one natural way of *construing* logical consequence[25] is a no-countermodels multiple-universe concept, one explication of which is given in the Church *Introduction to Mathematical Logic* [5]. In the pre-WWII period, the expression 'logical consequence' was ambiguous. Tarski [63] was mistaken when he repeatedly writes 'the concept of logical consequence', 'the concept of consequence', 'the common concept of consequence', 'the consequence relation', and so on, suggesting that, aside from a little vagueness, there is essentially only one concept expressed in the then-extant literature by the noun phrase 'logical consequence'. It is probably significant that Tarski does not mention Hilbert's *Foundations of Geometry* [32] in the 1936 consequence-definition paper, nor, for that matter, in the entire *Logic, Semantics, Metamathematics: Papers from 1923 to 1938*, nor even in the

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[24]Of course, Church's pluralistic tendencies might have been formed when he was studying with his teacher and dissertation supervisor Oswald Veblen, the American Postulate Theorist, before he studied Frege.

[25]The idea that logicians down through the history of logic were expressing a Tarskian no-countermodels concept by the phrase 'is a consequence of is implausible to say the least. Tarski's stated goal was not to characterize the traditional concept but merely to define 'a new concept which coincided in extent with the common one' [63, p. 409].

Introduction to Logic and to the Methodology of Deductive Sciences [66].[26] Nevertheless, as noted above, there is ample evidence that Tarski was aware of the pluralistic framework and its conception of multiple ranges for individual variables ([59], [70, pp. 199–209] to cite one prominent passage). Perhaps surprisingly, he nowhere shows awareness that in presenting the monistic fixed-universe conception he might be taken as rejecting the pluralistic multiple-universe conception. But, as far as we know there in not a scintilla of direct evidence relevant to determining his process of deliberation.

What difference does it make? If we define a tautology as a proposition that is a logical consequence of any and every set of propositions, then a tautology is a consequence both of a given proposition and of its negation, and it is a consequence of the null set. If we define a contradiction as a proposition that has as a consequence any and every proposition, then a contradiction has as a consequence the negation of each of its consequences, indeed its own negation. Under these two definitions, a large chasm between the two definitions of 'logical consequence' opens. In the case of the multiple-universe view the only pure cardinality propositions that are contradictory are those to the effect that the universe is empty and the only ones that are tautological are those to the effect that the universe is non-empty. By a pure cardinality proposition is meant one to the effect that, for a certain cardinal number, the universe does (or does not) have at least, exactly, or at most that number of members. In the case of the fixed-universe view, every pure cardinality proposition is contradictory or tautological, a result that is hard to swallow.

5. Concluding remarks

> If you by your rules would measure what doth not with your rules agree, forgetting all your learning seek ye first what *its* rules may be. – Wagner, *Die Meistersinger.*

Why would Tarski base his consequence definition on the monistic fixed-universe framework? Before attempting to answer this question it is important to note that we are not asking why Tarski would make the monistic fixed-universe *choice*. We have no evidence that any such choice was ever made. The question we ask is how the monistic view came to play a foundation role in Tarski's consequence-definition paper [63]. Probably, the most important consideration is that in his earliest logic training and in his dissertation *Principia Mathematica* was authoritative if not scriptural.

Once the monistic view was in place what obstacles may have kept Tarski from questioning it? For one thing, Tarski was never bothered by the limiting cases of tautologies and contradictions, or by the question of what human faculty is needed to determine their truth or falsity. Moreover, Tarski never seemed to

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[26]However, Hilbert's *Foundations of Geometry* [32] is mentioned in the second edition of *Logic, Semantics, Metamathematics* [70], but not by Tarski – it is on p. xvii of the Editor's Introduction.

have appreciated the De Morgan-Boole discovery of independent discourses each with its own limited universe of discourse – a discovery that would come to demote what others had seen as 'the universe of individuals' – from an exalted and unique place, a *sui generis*, to being just one of many universes of discourse (and a most questionable one at that). Some scholars have argued that Tarski had more deliberate reasons for leaning toward a monistic view (Rodriguez-Consuegra [48], Corcoran [22]). Although we and many others believe Tarski did not question the monistic, fixed individual-variable range view, for completeness it should be said that some well-informed scholars think that Tarski had not only questioned it but actually rejected it. They think that its presence in the consequence-definition paper is to be attributed to space limitation, style, rhetoric, pedagogy, and other contingent considerations. Thus, they allow such contingencies to outweigh truth in Tarski's deliberations. For example, David Hitchcock wrote the following (personal communication).

> It seems quite puzzling that Tarski did not allow variation of the range of variables in his 1936 logical consequence paper, given (1) the extensive treatment in his truth monograph ([59], [70, pp. 199–209]) of the concept of a truth in a domain, (2) the fact that the logical consequence paper is an application of ideas from the truth monograph, and (3) the fact that the fixity of the domain overgenerates consequences by (for example) making the existence of at least two individuals a consequence of the existence of at least one individual. The following possible explanation occurs to us. The paragraph in brackets on p. 415 of Tarski [70] suggests that Tarski simplified the exposition of his ideas, as would be appropriate for an audience of philosophers who were not necessarily mathematically sophisticated. He may have consciously avoided exploration of the variable-domain alternative in his 1936 logical consequence paper, in order to keep things simple for his audience. As far as I can see, there is no mention of a domain-relative conception of truth in the other paper that Tarski presented at the September 1935 conference, 'The establishment of scientific semantics', evidently for the same reason of keeping things simple; see the remark at the bottom of page 406 of Tarski [70].

This is in keeping with what Hitchcock and Stroińska published [33, p. 170].

It is hard to imagine a motivation for promulgating revisionist history that would make Tarski's pre-WWII, Warsaw-period thinking agree with his (and the dominant) post-WWII thinking about logical consequence, unless it is simply an inability to accept the fact that even Tarski can change his mind. One dramatic example of the evolution of his thought is his change from – at first – working in higher-order frameworks before WWII to – then later – regarding them as wrongheaded: in the late 1970s he bluntly told Corcoran that the definitional equivalence

results in Corcoran, Frank, and Maloney [23] were 'meaningless' because they concerned theories with second-order underlying logics (personal communication).[27] By 1936 Tarski had not yet fully shifted out of the monistic, Frege-Russell, fixed-universe paradigm that had been presupposed in his Warsaw training. Between his Warsaw period and his Berkeley period, Tarski's philosophy underwent many other radical changes. Not only was higher-order logic banned and replaced by first-order logic, but also type theory gave way to set theory, categoricity was de-emphasized in favor of decidability and deductive completeness, and the prominence of propositional logic was weakened while that of equational logic strengthened (e.g., Tarski-Givant [75]).

In one of his last sessions with Tarski, Corcoran asked him whether his research had caused him to change his mind over the years on fundamental issues. He said in immediate response that it would be sad if there were no cases of this – but then, after a long silence, he asked whether Corcoran had had anything in particular in mind. When Corcoran said no, Tarski said he would give it some thought. The topic never came up again (personal communication).

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[27]Tarski's oral evaluation contrasts with the evaluation published by Haskell Curry in Mathematical Reviews [24].

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