# Tomorrow's sea-battle and the beginning of temporal logic

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**Abstract.** Jan Łukasiewicz's consideration of the deterministic consequences of the law of the excluded middle and the principle of causality gave the incentive to the development of temporal logic. Formal reformulation of arguments in favour of determinism is possible in the language of temporal logic. Following Jan Łukasiewicz both the arguments, the argument from the law of the excluded middle and the argument from the principle of causality, will be discussed concomitantly.

**Keywords.** tomorrow's sea-battle, temporal logic.

## 1. Introduction

Aristotle (384 BC–322 BC) in his famous tomorrow's sea-battle passage of *De Interpretatione* 9 $19^a$ 30 stated the problem of the logical value of statements about future contingencies. The problem has been discussed since the age of Aristotle as an important philosophical question in connection with determinism. There are many solutions to it [1]. Diodorus Cronus from the Megarian school of philosophy is famous as the author of a version of the problem in his notorious Master Argument. In this subject some achievements are due to Avicenna (980–1037). His work influenced medieval logicians Albertus Magnus (1193/1206–1280) and William of Ockham (c. 1288–c. 1348). In the 19th century Charles Sanders Peirce (1839–1914) wrote that he did not share the common opinion that time is an extralogical matter. The deterministic consequence of the law of the excluded-middle (LEM) and the principle of causality (PC) were considered by Jan Łukasiewicz (1878–1956). It was, as writes Słupecki: [8, Introduction, p. vii]

> ... the problem in which Łukasiewicz was most interested almost all his life and which he strove to solve with extraordinary effort and passion was the problem of determinism. It inspired him with the most brilliant idea, that of many-valued logics.

To avoid fatalism, the deterministic (PRE-DET) consequence of LEM, he proposed abolishing the logical principle of bivalence (PB — every proposition is either true or false) — as Łukasiewicz called it [5] — and to introduce a new logical value [6, 7, 9]. He laid the ground for the historically first elaboration of three-valued logic [4]. The sentences about future contingencies would be neither true nor false; neither 1 nor 0. The logical value of such sentences would be intermediate. The inventor of temporal logic, Arthur Norman Prior (1914-1969), was concerned with the philosophical matters of free will and predestination. He learned logic from Polish logicians, in particular Jan Łukasiewicz (even using his parenthesis-free notation); in his early tense-logical considerations he referred to many-valued logic of Łukasiewicz [11, 12, 13, 14, 15, 16, 17, 18]. Many-Valued approaches to temporal logic were also discussed by Rescher and Urquhart [19, pp. 213–237]. Though attempts at formalizing future contingents using many-valued logics were not successful, the idea of many valuedness in temporal logic is still fruitful in particular in *AI* [1, Łuksiewicz's contribution to temporal logic, 149–154] and model checking [3].

In Łukasiewicz's opinion [6, p. 126]:

> A trivalent system of logic ... differs from ordinary bivalent logic, that only one known so far, as much as non-Euclidean systems of geometry differ from Euclidean geometry.

For him [10, pp. 84–85] it contributes to the

> struggle for the liberation of the human spirit [from the] logical coercion given by Aristotle's science as a system of principles and theorems connected by logical relationships. [...] The creative mind revolts against this concept of science [...] A system of three-valued logic [...] destroys the former concept of science, based on necessity [...] Logic is a free product of man, like a work of art.

The thought that many-valuedness is connected with freedom is still vivid. For Karpenko [2, pp. 143–144]

> ...Łukasiewicz finite-valued logic, are not restrictions (as they are usually viewed), but in a sense, extensions of $\mathbf{C}_2$[the classical propositional logic]. The repercussions of such an extension are quite serious as well rather surprising. The extension of the very basic logical universe resulted in the logics of continual nature; in the possibility to characterize, structure, and describe classes of prime numbers. Are all of these required for the logical reasoning? On the other hand, the problem of fatalism and free will is also of continual nature, which usually goes unnoticed. As we wrote in this book for the refutation of the doctrine of logical fatalism, Łukasiewicz, without being aware of it, abandoned discreteness for continuity.

Referring to his book Karpenko even ask [2, p. 4]:

> Is there, however, any link between the doctrine of logical fatalism and prime numbers?

In contemporary temporal logic the discussion of PRE-DET is usually limited to logical PRE-DET. Łukasiewicz also considered the argument from PC, i.e. the principle that any temporal fact has its cause in another fact that occurs at an earlier moment.

Usually the argument from PC in favour of PRE-DET is assumed as evident. Łukasiewicz questioned the argument and showed that both the arguments from LEM and PC are independent of one another. Moreover, Łuksiewicz questioned what in temporal logic discussions is treated almost as dogma, that the past is completely determined (POST-DET): We should not treat the past differently from the future [8, p. 127]. He rejects the Latin saying "facta infecta fieri non possunt" that is, what once has happened cannot become not happened. In order to analyze POST-DET, the principle of effectivity (PE), as a principle symmetrical to PC, should be considered.

## 2. Language and semantics

To use a formal logic to solve a philosophical problem, we have to have:

1. a formal language in that the problem can be expressed in an intuitively satisfactory way,
2. the logic should be neutral with respect to this problem, i.e. the formula that expresses it should not be a thesis of the logic (an analytical truth of the language).

The thesis of determinism as consisting of two theses, the thesis of pre-determinism PRE-DET and the thesis of post-determinism POST-DET, can be formulated as follows:

DET. If $\varphi$, then

- PRE-DET. at any earlier moment it was true that there would be $\varphi$

  and
- POST-DET. at any later moment it will be true that there was $\varphi$

The principle of causality says:

- PC. If $\varphi$ occurs at $t$, then at $t_1$, some moment earlier than $t$, and at any moment between $t$ and $t_1$ it was true that there will be $\varphi$.

The principle of effectivity, as symmetrical to PC, may be formulated as follows:

- PE. If $\varphi$ occurs at $t$, then at $t_1$, some moment later than $t$, and at any moment between $t$ and $t_1$ it will be true that there was $\varphi$.

The relation of causality is transitive [9, p. 28]:

> This means that for any facts $\phi$, $\psi$, and $\chi$, if $\phi$ is the cause of $\psi$ and $\psi$ is the cause of $\chi$, then $\phi$ is the cause of $\chi$.

Both the theses of pre- and post-determinism, PRE-DET and POST-DET, and both the principles of causality and effectivity, PC and PE, can be expressed in the language of temporal logic.

Let the language consist of:

1. $p_1, p_2, \ldots$ — propositional letters,
2. a functionally complete set of classical propositional connectives,
3. temporal operators (past tense and future tense operators).

Let $AP$ (Atomic Propositions) be the set of propositional letters. Formulas are defined in the usual way and will be denoted by Greek letters: $\varphi, \psi, \ldots$, if necessary with indices.

Let time be $\mathfrak{T} = \langle T, < \rangle$, where $T$ is a non-empty set (of moments) and $<$ is a binary (earlier-later) relation on $T$. No conditions on $<$ are imposed.

The temporal world $W$ consists of time $\langle T, < \rangle$ and facts that occur at elements of $T$. Let $V$ be a function, valuation, that to each $t$ ($\in T$) assigns a subset of $AP$, the set of propositional letters that are true at this point, $V \colon T \to 2^{AP}$.

Let temporal operators be defined in the usual way, i.e. as follows:

**Definition 2.1** ($G$)**.** $\langle T, <, V \rangle, t \models G\varphi$ iff for any $t_1, t < t_1 : \langle T, <, V \rangle, t_1 \models \varphi$.

**Definition 2.2** ($F$)**.** $\langle T, <, V \rangle, t \models F\varphi$ iff there is $t_1, t < t_1 : \langle T, <, V \rangle, t_1 \models \varphi$.

**Definition 2.3** ($H$)**.** $\langle T, <, V \rangle, t \models H\varphi$ iff for any $t_1, t_1 < t : \langle T, <, V \rangle, t_1 \models \varphi$.

**Definition 2.4** ($P$)**.** $\langle T, <, V \rangle, t \models P\varphi$ iff there is $t_1, t_1 < t : \langle T, <, V \rangle, t_1 \models \varphi$.

Operators $G, H$ are dual to, respectively, $F$ and $P$:

- $G\varphi \leftrightarrow \neg F \neg \varphi$,
- $H\varphi \leftrightarrow \neg P \neg \varphi$.

Let us call the language Priorean. Let $K_t$ be the set of all formulas that are satisfied in any model of whatever are the set of moments of time $T$, the relation $<$ and the valuation $V$, $\varphi \in K_t$ iff for any $W \colon W \models \varphi$.

The operators are usually read as:

$F$ — it will at some time be the case that …
$P$ — it has at some time been the case that …
$H$ — it has always been the case that …
$G$ — it will always be the case that …

Formulas of the language are recursively defined by:

$$\varphi ::= p | \neg\varphi | \varphi \vee \varphi | \varphi \wedge \varphi | \varphi \to \varphi | F\varphi | P\varphi | G\varphi | H\varphi$$

In Priorean language PRE-DET and POST-DET are expressed as follows:

- PRE-DET. $\varphi \to HF\varphi$
- POST-DET. $\varphi \to GP\varphi$

Both the classes of formulas PRE-DET and POST-DET are theses of $K_t$ [19, Chapter VI], the minimal logic of the Priorean language, i.e. they are propositions that are satisfied in any model independently of the property of time. Thus the Priorean language does not fulfil the condition of neutrality.

In the following two languages it will be considered in which:

- the rejection of both the theses PRE-DET and POST-DET will be possible, and
- the principles of PC and PE can be expressed, and
- LEM will hold.

## 3. Branching time logic

Since the thesis of determinism, DET, is a thesis of minimal $K_t$ logic, a new language has to be defined. Ockamist and Peircean languages are well known proposals. In any case it is assumed that time is branching in the future with many possible time-lines (histories). There are many courses of events possible. Since we are interested in neutral language, i.e. a language such that no discussed thesis is a thesis of minimal logic of the language, we do allow also branching in the past.

A branch of $\mathfrak{T}$ is any maximal linearly ordered subset of $T$. Each branch represents a possible course of events. For a point $t$ and branch $b$, if $t$ is a member of $b$, we say that $t$ lies on $b$ or that $b$ goes through $t$.

To say that at point $t$ there will be $\varphi$ means, that at any branch that goes through $t$ there will be $\varphi$. Analogously the past tense operator is understood. Let $B(t)$ be the set of all branches that go through $t$.

**Definition 3.1** ($F_\Box$)**.** $\langle T, <, V\rangle,\ t \models F_\Box\varphi$ iff for any $b \in B(t)$ there is $t_1 \in b,\ t < t_1$ : $\langle T, <, V\rangle, t_1 \models \varphi$.

**Definition 3.2** ($P_\Box$)**.** $\langle T, <, V\rangle,\ t \models P_\Box\varphi$ iff for any $b \in B(t)$ there is $t_1 \in b,\ t_1 < t$ : $\langle T, <, V\rangle, t_1 \models \varphi$.

The formulations of definitions of $G$ and $H$ do not differ of respective definitions of the Priorean language.

The theses PRE-DET and POST-DET are expressible as:

- PRE-DET. $\varphi \to HF_\Box\varphi$
- POST-DET. $\varphi \to GP_\Box\varphi$

Neither $p \to HF_\Box p$ nor $p \to GP_\Box p$ are theses of the minimal logic of the language of branching time.

If PC is valid, then

- $\varphi \to P_\Box F_\Box\varphi$

should be valid.

If PE is valid, then

- $\varphi \to F_\Box P_\Box\varphi$

should be valid.

It could be remarked that neither formula expresses the fact that the cause/effect is in any "between" moment.

Neither $p \to P_\Box F_\Box p$ nor $p \to F_\Box P_\Box p$ are theses of the minimal logic of the language of branching time.

**FS:** for any $t$ and for any $t_1,\ t_1 \leq t$:
1. there exists $t_2$ such that $t_2 < t_1$, and
2. for any $t_3$: if $t_2 < t_3$, then $t_3 < t_1$ or $t_1 \leq t_3 \leq t$ or $t < t_3$.

**PS:** for any $t$ and for any $t_1,\ t \leq t_1$:
1. there exists $t_2$ such that $t_1 < t_2$, and
2. for any $t_3$: if $t_3 < t_2$, then $t_1 < t_3$ or $t \leq t_3 \leq t_1$ or $t_3 < t$.

The conditions FS and PS taken jointly characterize both directions open segment $s$. Openness means that for any $t \in s$:

1. there is $t_1 \in s$ such that $t_1 < t$, and
2. there is $t_1 \in s$ such that $t < t_1$.

If $<$ is dense, $s$ can be finite, i.e.:

1. there is $t \in T$ such that for any $t_1 \in s$: $t < t_1$, and
2. there is $t \in T$ such that for any $t_1 \in s$: $t_1 < t$.

We maintain that PC holds if FS holds and PE holds if PS holds. It is true also when the relation of causality is transitive. Moreover, even if both the conditions are fulfilled, there are counter-models of PRE-DET and POST-DET. For any combination of PRE-DET, POST-DET, PC and PE there is a frame in which this combination is valid and for the other thesis or principle there is a counter-model. For example, for all PRE-DET, POST-DET, PC and PE there is a counter-model if time is branching in the past and in the future and the relation $<$ is discrete and irreflexive.

There are some objections to proposed semantics of branching time logic as a tool of formalizing arguments in favour of determinism. The relation of reachability of possible states of events is defined as the relation $<$ of earlier-later, a relation on the set of time points. Maybe it is supported by the idea of time as a result of change offered by the theory of the relativity of time. In the definition of the operators $F_{\Box}$ and $P_{\Box}$ any points of the branches are allowed. Due to the fact the formula $F\varphi$ rather expresses inevitability of $\varphi$ than its determination. Analogously it is true in the case of $P_{\Box}$. It seems that time should be the same independently of branch.

## 4. Temporal logic of possible worlds

Let us look for a solution such that the possibility of some event is referred to one and the same point of time, not — as it was the case in branching time logic — different moments are allowed. „There will be a sea-battle tomorrow" is true if it is determined that tomorrow there will be a sea-battle, i.e. in any possible course of events it will be tomorrow but not, e.g. in one possible course of events tomorrow, and another course the day after tomorrow.

The language of temporal logic of possible world not only will better expresses the idea of determination but it seems to be, in some sense, more typical for modal logics.

The relation of accessibility $\triangleleft$ will be a relation defined on sets of pairs consisting of a possible world and points of time. Possible worlds will not differ in time $\langle T, <\rangle$. They will differ only in valuation $V$.

Let $\mathfrak{W}(T, <) = \{\langle T, <, i\rangle : i \in I\}$. The relation $\triangleleft$ between $(W, t)$ and $(W_1, t_1)$ says that the moment $t_1 \in T$ of the world $W_1$ is reachable from a moment $t$ of the world $W$. Formally:

$$\triangleleft \subseteq (W \times T) \times (W_1 \times T).$$

For intuitive reason it will be assumed that for any $W, W_1 \in \mathfrak{W}$, and for any $t, t_1 \in T$:

- $(W, t) \triangleleft (W, t)$, i.e. $\triangleleft$ is reflexive

- $(W, t) \lhd (W_1, t_1)$ only if $t \leq t_1$

Let us define the tense operators.

**Definition 4.1** ($G_\lhd$). $W, t \models G_\lhd\varphi$ iff for any $t_1$, $t < t_1$, and for any $(W_1, t_1)$, $(W, t) \lhd (W_1, t_1)$: $W_1$, $t_1 \models \varphi$

**Definition 4.2** ($F_\lhd$). $W, t \models F_\lhd\varphi$ iff there is $t_1$, $t < t_1$, such that for any $(W_1, t_1)$, $(W, t) \lhd (W_1, t_1)$: $W_1$, $t_1 \models \varphi$

**Definition 4.3** ($H_\lhd$). $W, t \models H_\lhd\varphi$ iff for any $t_1$, $t_1 < t$, and for any $(W_1, t_1)$, $(W, t) \lhd (W_1, t_1)$: $W_1$, $t_1 \models \varphi$

**Definition 4.4** ($P_\lhd$). $W, t \models P_\lhd\varphi$ iff there is $t_1$, $t_1 < t$, such that for any $(W_1, t_1)$, $(W, t) \lhd (W_1, t_)$: $W_1$, $t_1 \models \varphi$

The language is defined in the usual way. The theses of PRE-DET and POST-DET and the principles of PC and PE are expressible as:

- PRE-DET. $\varphi \to H_\lhd F_\lhd\varphi$
- POST-DET. $\varphi \to G_\lhd P_\lhd\varphi$
- PC. $\varphi \to P_\lhd F_\lhd\varphi$
- PE. $\varphi \to F_\lhd P_\lhd\varphi$

PC and PE are not theses of the minimal logic of the language. They hold if some conditions on the relation $\lhd$ are imposed.

**FS1:** for any $W, t, t_1$: if $(W, t_1) \lhd (W, t)$, then there exists
1. $t_2$ such that $t_2 < t_1$, and
2. for any $W_1, t_3$: if $(W, t_2) \lhd (W_1, t_3)$, and $t_3 \leq t$, then $W_1 = W$

**PS1:** for any $W, t, t_1$: if $(W, t) \lhd (W, t_1)$, then there exists
1. $t_2$ such that $t_1 < t_2$, and
2. for any $W_1, t_3$: if $(W, t_3) \lhd (W_1, t_2)$, and $t \leq t_3$, then $W_1 = W$

In the frame $\langle T, <, \lhd \rangle$ that fulfils the condition $FS1$, PC holds. If $PS1$ is fulfilled, PE is valid. In the case the frame fulfils both the conditions $FS1$ and $PS1$, PC as well PE are valid. In any case a construction of a counter-model for the theses PRE-DET or POST-DET is possible. Using discussed semantics it is possible — as it was in the case of branching time logic — to constructed frames in which any combination and only that of PRE-DET, POST-DET, PC, PE is valid.

## 5. Conclusion

Abolishing arguments in favour of determinism does not mean that the universe, and we as part of it, is not determined. Newtonian physics gives reasons for Laplacean determinism.

Quantum physics says that everything happens with probability. Though not all of the interpretations of quantum physics imply indeterministic 'choices' of events. A universe governed by deterministic laws is preferably for the sake of making predictions. To be free means to make decisions according to mind. To make rational decisions, knowledge about a state of affairs is needed. According to quantum theory any attempt to know the state of affairs is changing the state. Our acts have only probable effects. To be free we have to grasp the reality. Logic is the fundamental part of our consciousness. Maybe a candidate for a logic of free will is the quantum logic.

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